ELEMENTARY CHAPTERS

IN

ASTRONOMY;

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M.DCCC.L.

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ASTRONOMY,

FROM THE "TRAITÉ ÉLÉMENTAIRE D'ASTRONOMIE PHYSIQUE" OF BIOT.

EDITED BY THE

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THE following pages contain a translation of the first five chapters of the last edition of Biot's *Traitè Elementaire d'Astronomie Physique*. They were translated with a different intention, but the admirable precision and clearness of description which characterize them led me to think that the publication of them would make a useful addition to our present list of elementary books. The chapters include so much of astronomy as can be conveniently learned by observation, without having recourse to the refinement of modern methods, and exhibit the manner in which ancient astronomers actually arrived at their knowledge; and I think that considerable advantages, in addition to the historical interest, attach to the mode which M. Biot has adopted of introducing the student to more complex methods. Those who have time and opportunity may be safely recommended to the original work as a very complete and admirable treatise. The present little extract is intended for those who have neither time nor opportunity, and I have taken care so to edit it that there shall be nothing to deter from its perusal those who compete for the lowest Mathematical Honours.

I am of opinion that some other portions of the same work might be with advantage adapted for Cambridge students; and shall be willing to give some time and attention to the preparation of further extracts in the event of the present publication meeting with approval.

CAMBRIDGE,
February, 1850.

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CHAPTER I.

ON THE GENERAL APPEARANCE OF THE HEAVENS.

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1. LET us suppose ourselves placed on an elevated spot, in an open country, where the view is unbroken on all sides. The *sun* has just *set;* but the part of the heavens where it has gone down is still illuminated by its rays. By degrees this light grows feeble, the darkness increases, *night* comes on, and the heavens, extended above our heads, appear to be a vault studded with a multitude of twinkling points; these are the *stars,* which the brightness of the sunlight prevented us from seeing during the *day.* The order, the arrangement of these heavenly bodies, appears fixed and unchangeable. It is the same to-day as it was in the most remote times. The configurations of the various groups of stars are still the same as the ancients described them to be, when they classed them under the name of *constellations,* and connected them, to assist the memory, with the figures of men or of animals. But these stars, subject as they are to a constant order, move all together in the heaven as by a general rotation, the effects of which we are not long in recognizing. Some sink towards the *west,* on the side where the sun disappeared; soon they set and disappear like the sun before them; whilst on the opposite side, towards the *east,* other stars *rise* and appear to come from below the *horizon,* that is, the points of the earth and of the sea which bound our view*. After having risen in the heavens to different *apparent altitudes,* they will descend again, setting in their turn, in the same manner as those which preceded them. But if in Europe we place ourselves in such a

* The various words which are here printed in italics, have in Astronomy a precise sense which cannot as yet be defined. They are used here with all the vagueness which attaches to their ordinary use, for the sake of describing the first impressions of the phenomena. Their scientific value will be settled afterwards; and in fact that value has generally not been defined until long after the terms have been in popular use.

manner as to have the east on the right hand, and the west on the left, we see in that portion of the heavens which faces us, and which is called the *north*, groups of stars which never set. Such is, for example, the constellation of the Great Bear, or Charles' Wain, with which every one is familiar. This constellation, and many of those which are found in the same part of the heavens, only disappear when they are extinguished by the too great glare of sunlight. We can see them through the whole of the night, and follow them down to the lowest point of their course, for they never reach the horizon. Observing them at different times throughout the night, we see them assume exactly reverse positions in the heavens, a natural consequence of that rotation which is common to them with all the other stars; and the centre of their motion, as indicated by these phenomena, appears to be a point situated towards the northern part of the heavens. But soon the heavens brighten towards the east; the light becomes strong enough to render invisible the stars which have just risen on that side: the west alone remains still dark; it is exactly the reverse of what took place at the beginning of night. The light continuing to increase, the stars grow gradually fainter; at length they become invisible, and the day opens upon all objects. The sun reappears; it rises in its turn in the east, like the other stars; it ascends, passes through the vault of heaven, then sinks, and disappears, or sets on the opposite side: then all the phenomena of night recommence in the same order, and follow the same laws.

The *moon*, of which we have not yet spoken, and which is remarkable on account of the magnitude of its disk, its brightness, and the changes which it undergoes, in respect of the form of the luminous part, changes which we call its *phases*, presents also analogous phenomena.

This motion of revolution, which is common to all the stars, and which is accomplished in the interval of a day and a night, is called the *diurnal motion.*

2. Since the stars lying towards the north, near the pivot or *pole*, round which the general motion takes place, remain always high above the horizon, whilst others, at a greater distance from this point, descend more nearly to the horizon, and others at a still greater distance sink below the horizon

altogether, we conclude that their *setting* is the effect of the magnitude of the circles which they describe, and that they complete them under the horizon, below the earth, when they disappear from our eyes. The most obvious analogy leads us to extend this explanation to stars situated in that part of the heavens which is opposite to the north, and which we call the *south.* These stars, after setting, also complete their revolution below the earth, to reappear, like the sun, in the east. If then, for distinctness of conception, we conceive a mathematical line, or *axis of rotation,* round which all this motion takes place, it will be necessary to conceive, that in Europe this axis appears elevated towards the north, and oblique to our horizon, that is to say, to the plane which, passing through our eyes and touching the earth's surface, separates the visible portion of the heavens from that which is concealed from us.

It is not necessary to represent this axis as anything material, really existing in space; it is only a geometrical conception, which defines the series of points in space which appear fixed in the midst of the general motion. It is the same with the *circles* which the stars appear to describe round this axis of their diurnal revolution; we are only to understand by these the points of space which we perceive them successively to occupy. As to the supposition that this diurnal motion is really and exactly circular, we shall employ it here merely as an hypothesis which expresses the appearances; for, although it is true in all geometrical strictness, the complete proof can only be given by means of very precise measurements, and by the help of considerations which cannot be introduced at present.

3. If we examine the heavens during a great number of nights, we notice certain heavenly bodies which change their places amongst the stars; these do not always form part of the same constellations: each day they approach some, and become more distant from others by an almost imperceptible quantity. This phenomenon is much more manifest in the case of the moon; for the relative position of that body varies, very sensibly, in the course of a single night. But, even for the other bodies of which we are speaking, their small displacements become at length perceptible, and carry them at length into entirely different parts of the heavens. On this

account they are called *planets,* that is to say, wandering stars, in opposition to the rest, which are, or which at first sight appear to be, relatively immoveable. These are called *fixed* stars, to express the relative permanence of their positions.

The planets hitherto known are in number sixteen; they are named as follows: Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune, Ceres, Pallas, Juno, Vesta, Astræa, Iris, Metis, Hebe, and Flora. The first five are visible to the naked eye; they have been known from the most remote antiquity. Uranus, discovered more recently, may perhaps be seen by a very acute eye; but Neptune never without the aid of a *telescope.* The remaining nine are so small, that only very excellent instruments will render them visible, and they are called *telescopic planets.*

The motions of the planets amongst the fixed stars are called *proper motions;* the moon and the sun have also proper motions, which are recognized in the same manner; that of the sun is especially remarkable on account of the phenomena to which it gives rise.

4. To make this appear, observe the sun for a number of days in succession when it is on the point of setting; and when it is below the horizon examine the stars which follow it and which set immediately after it; they will be easily remembered by the figures which they form in the heavens. After some days you will see these stars no longer; there will be other stars which will follow the sun and will set immediately after it. These same stars, on the preceding days, did not set until a long time after the sun: this body then has advanced towards them *from west towards east,* that is, in the direction opposite to that of the diurnal motion. In like manner, if you observe the heavens in the morning, some moments before sunrise, you will see the same appearances in the reverse order. The stars which rise to-day at the same time, or almost at the same time, as the sun, will, after some days, rise considerably before it. They will seem to move away from the sun, going from the east westward; or, which comes to the same thing, the sun will move from them from the west eastward; for it is more simple to suppose the sun to have a proper motion, than to suppose a general motion with reference to the sun common to all the stars. In con-

sequence of this proper motion, the sun appears to move through the whole circuit of the heavens, going from west towards east.

5. Again, in examining the heavens by night, at different seasons of the year, we find their face altogether changed. There are no longer the same stars visible; they are arranged and disposed differently. This is a very simple consequence of the sun's proper motion. In that portion of the heavens where the sun happens to be, the brightness of its light prevents us from seeing the stars with the naked eye; with telescopes it is possible to see them even in the day-time. But, as the sun leaves these stars in consequence of its proper motion, as it constantly travels eastward, they arrive above the horizon during the night and so become visible. We may see in this circumstance already the inaccuracy of the rough notions to which the first sight of the phenomena gives rise, and in virtue of which we should be led to believe that the heavens are divided into two portions which appear above the horizon successively, and of which one is occupied by stars, the other by the sun. Exact and continued observation of celestial phenomena will lead us to recognize many other illusions, and teach us to renounce them.

Those stars only which lie towards the north, and which never set, remain constantly visible in the heavens at night at all seasons of the year. But, at the same period of the night, we see them successively in different positions, according to their situation with reference to the sun; and in this way the proper motion of that body will be sensible even by reference to these stars.

6. The proper motion of the sun is not directed exactly from west to east, for every one is aware that at certain times the sun rises much more above our heads than at others, a fact which becomes very sensible in the variations of its heat, whence results the change of the *seasons*. It may be recognized also by noting the points of its rising and setting, which do not always coincide on the horizon with the same terrestrial objects. But the first proper motion of the sun, from west eastward, is the most considerable, since in virtue of that it makes the entire circuit of the heavens;

whereas the second motion, of which we are now speaking, seems to be confined between certain limits which the sun never exceeds. On the whole we have this result, that the sun describes in the heavens an oblique path which is not altogether directed from west to east, but which deviates from that direction only within certain determinate limits.

7. The proper motion of the moon, amongst the stars, is also directed, like that of the sun, from west to east, with variations much greater than in the case of the latter body. The proper motions of the planets also follow the same direction, in the greater part of their course. But it is found, that at certain determinate epochs, which are different for different planets, the proper motion diminishes by degrees, until at length it becomes altogether insensible. Then the planet appears *stationary* amongst the stars. After which its motion recommences, from east to west, that is to say, in the opposite direction to that of its previous motion; the result of which is that the planet appears to *retrograde* amongst the stars. But after some time this *retrogradation* diminishes; the planet stops, once more becoming *stationary*, then recommences its *direct* motion amongst the stars, that is from west to east. These phenomena, which have been observed from the most remote antiquity, are called the *stations* and *retrogradations* of the planets.

The proper motions of the planets, like that of the sun, are not directed exactly from west to east; they diverge from that direction within certain limits which they never pass. The ancient astronomers remarked that the five with which they were acquainted were always comprised within a narrow zone of the heavens, which they called the *Zodiac*. But the telescopic planets are not confined to these limits.

When we observe the planets with telescopes which magnify the visual angle which they subtend, and which angle we call their *apparent diameter*, they present the appearance of a rounded disk, such as would be presented by a body of spheroidal form viewed at a great distance. When this disk, by the effect of proper motion, is brought into the direction of any fixed star, it hides the star by intercepting the light which would have rendered it visible. This phenomenon, which is called an *occultation*, proves then that the planets are opaque bodies nearer to the earth than the fixed stars. The moon causes occultations of stars in this manner frequently, and sometimes even of planets. The moon therefore is nearer to us than these latter bodies, and like them opaque. It sometimes also intercepts the rays of the sun, and *eclipses* it either partially or totally. The stars, on the other hand, never present a sensible disk, even when viewed through the most powerful telescopes, which magnify the visual angle 1200 or 1500 times. This angle therefore is so small for the stars in consequence of the distance by which they are separated from us, that even so great a multiplication as this does not render it sensible; so that the image of the star, although so highly magnified, appears still only as a bright point, without any magnitude appreciable by the eye.

8. Furthermore, we discover from time to time in the heavens certain bodies which we have not perceived before; which in the first instance appear very minute, with little brightness, and are ordinarily accompanied by a kind of nebulosity or luminous *tail.* These bodies have also proper motions amongst the stars; but their paths are very variable, and they traverse the heavens in all directions. It is usually found that their brightness increases from the first moment of their appearance up to certain limits, after which it diminishes by like degrees; and finally, after a longer or shorter interval of time, they cease to be visible. The nebulosity, which almost always accompanies them, has caused them to receive the name of *comets,* or stars with long hair.

9. All the heavenly bodies of which we have spoken are treated of in the science which we call Astronomy. To observe and determine precisely their positions in the heavens, to follow their motions, to measure them with exactness, to ascertain the laws to which they are subject, and then to make use of these laws in order to predict their positions at any future time, or to assign those which they have occupied at any given past epoch,—such are the aims and ends of Astronomy.

10. Finally, we often see in the heavens certain bright meteors, the appearance of which lasts only for a few instants.

They become visible, at the point from which they appear to start, only at the moment in which they shoot from it; and they leave no permanent trace of themselves in that part of the heavens at which they disappear. Such are those globes of fire which sometimes shew themselves suddenly in space, followed by a bright tail, emitting brilliant sparks, and which, after a very rapid course of some instants in duration, often burst with a loud report. Such also are the meteors which are commonly known as *shooting-stars,* and which seem to have considerable connection with the preceding. These phenomena have been regarded for a long time as effects purely physical, produced by vapours existing in the air, and which accidentally take fire owing to causes unassigned; but, for some few years, very good reasons have been given for thinking that they also belong to the province of Astronomy. The full explanation of these reasons, as also of the most probable notions that have been formed relatively to the nature of these *meteors,* does not belong to this place. It will be sufficient here to observe, that the short duration of time in which they are visible is no good reason for excluding them from the catalogue of heavenly bodies. For at different epochs there have suddenly appeared shining amongst the fixed stars luminous points which had not been previously seen, which have appeared for a considerable time like stars in respect of their fixedness, and which after having passed through great variations in their lustre have at length ceased to be visible. Moreover, inequality of time of duration, or of visibility, is but an accidental characteristic of bodies; and though it may be employed to distinguish them, it can scarcely be made use of essentially to define them.

CHAPTER II.

ON THE FIGURE OF THE EARTH.

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11. We have now described the most obvious phenomena which the heavens present to an isolated observer; but amongst these phenomena, that of the rising and setting of the heavenly bodies is one of the most singular, and deserves our first attention. What is it then which conceals half of the heavens from us, and which we have called the *horizon?* is it the same for different countries? is it possible to reach it, and if so, what should we find beyond it?

All these questions, and many others, are easily resolved by reference to the experience of travellers, especially those who have made sea voyages. When persons at sea recede from the shore, they observe the hills and buildings to sink by degrees, until at length they seem to be swallowed up in the waters. This effect is not due merely to the distance, which causes objects to appear diminished in size; for when we lose sight of the land from the deck of a ship, it may still be seen from the masthead. In the meanwhile, the ship presents similar appearances to spectators on shore; they see it gradually sink, and at length disappear as if it had sunk into the waters, and precisely in the same manner as the sun when it sets. These phenomena, which may be observed always, and in all directions, prove distinctly that the surface of the sea is convex, and conceals distant objects from us by its convexity. For if the surface were plane, an isolated mountain, or even a tower standing upon it, would be always seen on all sides, unless the spectator should be at such a distance that the dimensions of the mountain should become insensible on account of the distance; but that could not happen except at very great distances indeed. The bases of lofty objects would not disappear sooner than the summits; they would not appear to sink gradually; and lastly, when they could no longer be seen from the deck of a vessel they would be equally invisible from the masthead.

The horizon of the sea then, which appears to terminate its surface, is not a real boundary, but an apparent boundary, relative to the actual position of the observer, and produced by the convexity of the surface of the waters. Those whom we watch as they leave the shore appear to us to go beyond it, but their horizon shifts with themselves. When they disappear from our view, if we ascend a hill on the sea-shore, we shall see again, for some time, the same vessel which appeared to have sunk in the waves.

It was a bold and important undertaking to determine what becomes of this apparent boundary, when we endeavour to reach it by going constantly in the same direction. Ferdinand Magellan, a Portuguese, was the first who realized this undertaking. He embarked from one of the Portuguese ports, and sailed westward. After a long voyage, he came to a great continent, already discovered by other navigators who had followed the same course; this was the continent of America. Not being able to find a passage in order to continue his voyage westward, he coasted the land going southward, arrived at its extremity, doubled it, and then found himself in a vast ocean already known, which we call the Pacific. He then pursued his course westward; after a considerable voyage he reached the Moluccas, and his ship going still westward, came at length again to Europe, and returned as though it had come from the east to the port from which it had sailed.

12. This memorable experiment, since repeated by a great number of navigators, proves that the total surface of the land and water is convex, and nowhere meeting the heavens. Moreover, to whatever country we may travel, we see always the general system of heavenly bodies revolving round the earth by the effect of the diurnal motion of the heavens.

13. Hence we must conclude that the heaven is not supported by the horizon of the sea, as at first sight one might suppose. This illusion arises from the fact that the sense of sight indicates to us only the actual existence of objects, in the direction of the visual rays which serve to render them perceptible to our eyes; and that in this case,

having nothing whatever to enable us to appreciate the inequality of distance, we involuntarily believe it to be zero. When the rays of light which come from a star graze the surface of the sea, it appears to us as though the star were upon the boundary of the surface. If we conceive a cone of visual rays, which shall have for its vertex the eye of the spectator and which shall touch the horizon of the sea, all points of the heavens situated upon these rays will appear to be contiguous to the surface of the water, as if the heavens rested upon it.

14. These results only enable us to conclude the roundness of the earth in one direction, namely, from west to east; it is equally sensible however from north to south, as is ascertained from the results of voyages made in that direction. Upon the land it is difficult to make this observation, because the horizon being almost always bounded by hills of greater or less elevation, we may be led to think that it is the hills which conceal from us whatever may be beyond. But we can complete these proofs by a more general consideration, which is applicable both to sea and land. This consideration is founded upon the fact that the same stars attain to different apparent altitudes above the horizon, as we change our own position on the earth's surface. For example, if we start from any place and go southward, we see the stars situated in that part of the heavens rise higher and higher above the horizon. The arcs which they describe in virtue of the diurnal motion are longer. Some which could not be seen at all from the spot from which we started begin to shew themselves. On the other hand the stars situated towards the north are depressed. Those which were always visible, but which described a pathway nearly reaching to the horizon, are now invisible in the lowest part of their course, which they complete below the horizon; precisely in the same manner as in going out to sea buildings and hills sink and disappear as we recede from them. The same phenomena occur in reverse order when we proceed from south to north. Thus, by changing our position on the earth's surface, and proceeding constantly from north southwards, or from south northwards, we can in a certain sense change our heavens. We can even make the *north pole* of the heavens sink

below the horizon, and another pole appear on the opposite side of the heavens, which we call the *south pole*. These phenomena give a great amount of additional evidence for the fact of the convexity of the earth's surface. The stars are in this case with reference to ourselves what the buildings and hills were with reference to the navigator leaving the sea-coast. The only difference is this, that the prospect of the navigator is unbounded on all sides, whereas on land this is not the case; a difference which compels us to have recourse to celestial marks to raise us above the obstacles lying on the earth's surface, which conceal its convexity from us. It is for the same reason that points most elevated above the earth's surface, as the summits of mountains and the tops of towers, receive first the light of the morning sun, and are illuminated by the last evening rays. By a necessary consequence, when the sun sets for a certain country, it appears at the height of its course for countries lying very far to the west; whilst it is rising to countries lying still beyond.

15. The inequalities which limit our view upon the actual surface of the earth have constrained us to have recourse to these great celestial marks in order to recognize its convexity. But very simple considerations of physical geography will lead us to the same conclusion. We have already established the fact that the general surface of the ocean is convex in all directions about a given point. This is so to speak proved to ocular demonstration by the circular form which is always presented to the eye of the sailor by the curve of contact, in which the earth's surface is touched by the cone of visual rays which enter the eye, and which determines the portion of the ocean which he can see. Now it is easy to prove that the solid and habitable surface of the earth differs everywhere very little from that of the water, of which it is (so to speak) only the continuation. For in the first place the continents are surrounded by seas which run up into them at a number of openings. Thus for example America is divided into two parts, which are only connected by a very narrow tongue of land. In like manner the old continent is separated into a great number of parts by many seas, such as the Mediterranean, the Red sea, the Black sea,

the Baltic, which are only ramifications of the ocean with which they communicate. No point in the interior of the continents is therefore very distant from the sea. We observe also that their coasts are in no part very much elevated above the level of the waters which wash them. It follows therefore of necessity that their surface must very nearly follow in form the convexity of the ocean.

This becomes still more evident, if we consider the course of the rivers by which the continents are intersected. Many of them, such as the Rhine, the Danube, the Volga, the Nile, the Amazon, pass through very considerable tracts of country. The Amazon alone runs over more than 1200 leagues, and receives many rivers which are 6 or 700 leagues in length. All those great rivers flow into the sea; none of them have very high banks. They indicate, therefore, by the slowness or quickness of their streams, the *fall* of the countries which they traverse, that is to say, the difference between their curvatures and that of the sea.

Moreover, it is easy to see that this fall is in general very slight; for all these rivers are navigable, and their stream becomes very slow near their mouths. Nature itself gives us a very good means of taking the level in one of its grandest phenomena. Twice, in each interval of a day and night, the ocean rises and falls through several feet, by a regular motion which we call the *flow* and *ebb* of the tide. The waters of the sea thus raised are forced up the rivers, and ascend to a considerable distance above their mouths. In the Amazon, for instance, they advance more than 200 leagues. It is demonstrated then by this fact that the fall of the rivers differs very little from the curvature of the ocean: whence it also follows that the convexity of the continents is nearly the same as that of the seas.

16. The round figure of the earth is still further manifested, and that in a very striking manner in many phenomena presented by the moon; but this requires some preliminary notions in order to become intelligible. We know that the moon undergoes, in the extent of the portion illuminated, very sensible variations to which we give the name of *phases*. It appears successively under the form of a *crescent*, of a *semicircle*, and of a complete *circle*, after which its disk

diminishes as gradually as it increased. These *periodic* variations, variations that is, which always follow each other in the same order, have such striking relations to the position of the sun, that we can certainly conclude that the moon is a round opaque body illuminated by the sun; the face of which turned towards us, sometimes bright, sometimes dark, or partly one and partly the other, according to the position of the sun, offers all the appearances which we observe. In fact, this hypothesis represents all the appearances so naturally that it becomes impossible to doubt its truth.

The moon, then, not being luminous itself, but shining by light borrowed from the sun, if it should happen that, by the effect of its proper motion, it should pass between that body and the earth, it is evident that it must conceal it from our view either wholly or in part. This is in fact precisely what takes place. The moon, under such circumstances, appears on the face of the sun like a black patch, and prevents us from seeing the sun, or at least intercepts a portion of its light. This phenomenon, which has been already mentioned in the general description given in the preceding chapter, is called an *eclipse of the sun.*

Sometimes also the moon becomes suddenly obscured; and in the course of a few hours, gradually loses and then regains its brightness. The boundary of its disk which disappears first is the first to reappear, precisely as would happen if an opaque body lighted by a candle should enter the shadow cast by some other body. This phenomenon, which is called an *eclipse of the moon,* never takes place when the moon appears wholly illuminated and opposite to the sun. It is natural to conclude that the earth, being illuminated on one side by the sun, casts behind it a shadow into which the moon enters when it is eclipsed.

It is the form of this shadow cast upon the moon's disk which indicates the roundness of the earth's figure. When the moon begins to enter it, the greatest part of the disk is still illuminated by the sun. This luminous portion is not terminated by a straight line, as would be the case if the boundary of the earth's shadow were rectilinear. It has the form of a crescent, the convexity of which is turned towards the illuminated portion of the moon. This convexity evidently indicates the roundness of the shadow, and consequently the roundness of the earth which casts it. This same appearance is reproduced when the moon begins to emerge from the earth's shadow.

17. Connecting the result of these observations with what we learn from maritime voyages, we may conclude with certainty that *the land and water together form a globular body isolated in space.*

18. Although this conclusion is very certain, being a logical deduction from well-established facts, there is some difficulty in conceiving of the earth as isolated and unsupported in space. This arises from our generalizing the notion of weight or gravity which we observe as affecting bodies situated near the earth's surface. It is not, however, a correct conclusion from this, that the earth must tend to one part of space rather than another. So that, when observation teaches us that the earth is actually unsupported, free and isolated, there is nothing in the phenomena presented by heavy bodies which ought to cause us any surprise.

19. Still further, since the earth is round, the various people who inhabit it have their heads turned to different parts of the heavens. There are people therefore who stand in a position diametrically opposed to our own, and having their feet turned towards ours. For this reason they are called *antipodes,* and each country has its own. This arrangement appears strange, but it is not the less real. These considerations are brought forward here in order to shew that we must not be astonished at new truths which observation and experiment reveal to us. The astonishment which they cause usually arises from our allowing ourselves to regard as universal those things to which we happen to be accustomed. This is a prejudice from which it is necessary to get free, and which is in fact gradually dissipated as we acquire the habit of observing.

For the rest, when we are more advanced in Astronomy, the round figure of the earth will appear to us in no way extraordinary. For, as has been already mentioned, when we observe the heavenly bodies with instruments which very much magnify their images, we remark in the case of several phenomena which prove that they also are round. Of this number are the sun, the moon, and the planets. If the same test, when applied to the stars does not succeed in exhibiting them as bodies of finite dimensions, this may very well be accounted for by their excessive distance; and there is nothing in it incompatible with the fact of their having a spheroidal figure. The roundness of the earth then, which appears so strange at first sight, is only a property which is common to it with many other bodies, isolated like itself in the infinite space of the heavens.

20. The earth being convex, the perpendiculars drawn to its surface at different points of its surface are not parallel; they converge within its surface. If they all met in one point, the earth would be spherical. Generally, the manner in which they are inclined to each other indicates the form of the curve surface. For if we conceive a flexible straight line AB, to which a number of lines Pp, $P'p'$, $P''p''$ are drawn perpendicular, at the points $MM'M''$, (fig. 1) then, so long as the line remains straight, the perpendiculars will be parallel to each other. But if the line be curved in one plane, as in fig. 2, the perpendiculars will approach each other towards the inner part of the curve, while they recede from each other on the opposite side; and this change of direction will be more marked as the curvature is more decided. The directions of these perpendiculars is therefore an element very necessary to be determined with reference to the earth's surface. And they are indicated at each place by the direction of motion which heavy bodies assume, when they are allowed to fall freely; for it is the result of observation that the descent of bodies falling freely is always perpendicular to the surface of still water, which indicates everywhere the form of the earth's surface, neglecting its irregularities.

21. As this fact is the basis of all the knowledge which we have acquired relative to the figure of the earth and the celestial motions, it is of importance to establish its truth by rigorous proofs. This determination will be the object of the following chapter.

CHAPTER III.

DETERMINATION OF THE DIRECTION IN WHICH GRAVITY ACTS AT EACH POINT OF THE EARTH'S SURFACE; DEFINITION OF THE VERTICAL LINE, AND OF THE HORIZONTAL PLANE. MEANS OF DETERMINING THEM EXPERIMENTALLY.

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22. WE denote by the term *gravity* the force which draws all bodies towards the earth's surface when they are unsupported. This force, exerted upon all the particles of bodies, constitutes their entire *weight*. Now experiment shews that this weight does not vary perceptibly, when we divide the heavy body, and resolve it into any number of parts however small. The total weight is therefore the sum of the elementary forces which act upon all the parts separately; and these forces act in directions sensibly parallel to each other. Also we observe that all bodies fall in directions sensibly parallel at the same place.

But the exactness of this parallelism may be proved experimentally by one of its physical consequences, with much more rigour than by direct observation. When parallel forces act upon all the points of a material system, it is easy to shew that they may be all compounded into one resultant, equal to their sum, and the direction of which passes through a certain point of the system which is called the *centre of parallel forces* *.

[* The position of this point may be found, exactly in the same manner as that of the centre of gravity is found in page 227 of Goodwin's *Course*, (3rd ed.). In fact, in that investigation it is only necessary to suppose the weights of the particles to be any parallel forces acting at those points; and by thus following the process there given, *mutatis mutandis*, we determine the position of a point at which the whole system of parallel forces may be supposed to act; but the quantities which fix this position will be independent of the particular direction in which the forces act, that is, they will be the same for any sets of parallel forces in which the forces are the same in magnitude and applied at the same points, whatever may be the directions of the forces; and the point thus determined is therefore called the *centre of parallel forces*, it being a point central not with regard to one set of parallel forces but with regard to all sets having the forces equal, and the points of application the same. In fact, a set of parallel forces

Then if gravity act according to this condition of parallelism, there ought to be in each body a like centre which we may call the *centre of gravity*, and which will be such that, if the body be supported at this one point by a force sufficient to counteract its weight, it will be in equilibrium. And, conversely, if it is in equilibrium when supported by a single force, we may be sure that the direction of that force passes through the centre of gravity of the body.

Now conceive a heavy body M (fig. 3), of any kind, but solid, that is to say, such that its parts naturally hold together; a piece of metal, for instance. Denote by G the point of its mass known or unknown, at which its centre of gravity is situated. At any point S of its surface fix a thread SF, very fine, flexible, but not extensible, or at least resisting extension sufficiently to be able to support the weight M without breaking. Let the other extremity F of the thread be attached to a fixed rigid support AB, which rests upon the surface of the earth; and having gently let go the body M, so as to stretch the thread, let it be left to itself in the lowest position which the inextensible thread will allow it to occupy. Experiment shews that the body M will settle itself into a position in which it will rest, so as to stretch the string in the direction of a certain straight line SF. The resistance of the fixed point F will then be in equilibrium with the weight of the body M. Also, since this resistance is transmitted only through the thread SF, which is stretched into a straight line, it follows that the centre of gravity G must be on the line FS produced, and that the direction of this line must coincide with that of the force of gravity which acts at G. Here then we have the means of actually exhibiting this latter direction with a precision proportional to the fineness of the thread SF. The apparatus just described is called the *plumb-line*.

Again, suppose that we suspend three similar threads A,B,D, a little distance from each other, as represented in fig. 4, giving to each a length of about two or three yards; and imagine their directions F_1G_1, F_2G_2, F_3G_3 to be produced

have not a *point* of application for their resultant, but a *line* of application. In elementary statical treatises it is usually assumed that the weights of the particles of a system may be considered as a system of parallel forces; in the text the properties of the centre of parallel forces are assumed in order to shew that gravity acts upon adjacent particles in parallel directions. Ed.]

indefinitely. If we endeavour to project by the eye the thread *A* upon the thread *B*, we shall see that they coincide accurately, and that one eclipses the other throughout its whole length, without any appreciable deviation; so that the two threads lie in one plane *AB*. The same mode of proof will shew that *A* and *D* are also in one plane *AD*; and likewise *D* and *B* in one plane *DB*; all with equal exactness. Moreover, if we imagine these three planes produced indefinitely, they can have only one common point *C*, which will be the point of intersection of their three common lines of intersection *A*, *B*, *D*, which coincide with the directions of the three threads. So far then as we can judge from this experiment, the three directions of gravity indicated by these threads, converge to the same point; or are parallel to each other, if this point is infinitely distant.

Now let us suspend another plumb-line *E*, at a little distance from *A* and *B*; we can prove in like manner that *A*, *B*, and *E*, intersect in a single point, which can be therefore no other than the point *C* in which the three former intersect. This reasoning being applied to any number of threads, it results that all the directions of gravity, observed thus in a space of moderate extent, converge towards the same point *C*, or are parallel, *as nearly as the senses can judge.*

This last restriction is indispensable; for whatever rigour may appear to belong to the preceding demonstration, it is still founded upon a physical appreciation, which has of necessity its limits. In the first place, if the point of intersection *C* be at a very great distance, we cannot in this experimental manner establish its existence, because the convergence of the threads, though real, may possibly not produce any appreciable variation in their mutual distances with lengths so small comparatively as those which we are able to give them. This is, in fact, what we find to be the case when we endeavour to measure this variation. But besides, it is possible that the threads, without being each two mathematically in the same plane, may nevertheless so nearly satisfy this condition, that the error may not be capable of being detected by the eye. It is necessary then to restrain the consequences of these experiments within the limits which this possibility of error assigns to them; and we shall do this by saying, that according to the indications given by them, *the directions of gravity,* *observed over a very small extent of the earth's surface, appear to be sensibly parallel to each other, or convergent to a very remote point.*

23. The direction of gravity, thus belonging to each particular point of the earth's surface, is called *the vertical at that place;* and it is determined, as has been said, by the direction of the plumb-line. If we conceive it indefinitely produced towards the heavens, and also towards the interior of the earth, the point above in which it would meet the apparent surface of the heavens is called *the zenith;* the point below is called *the nadir.* These two terms, like many others used in astronomy, are of Arabic origin; the Arabs having first instructed Europe in the ancient Greek astronomy, and transmitted to us its results.

Every plane passing through the vertical line is called a *vertical plane,* or shortly, *a vertical.*

24. It may be shewn that *the vertical,* thus determined, *is, at each place, normal to the surface of a still fluid.*

To prove this, it is necessary to make use of a theorem in optics, namely, that when a pencil of rays are incident from a point upon a polished plane surface, the reflected rays all proceed as from a point lying upon the normal to the surface from the point of incidence, and at the same distance from the surface as the point of incidence, but on the opposite side of it. (Goodwin's *Course,* p. 405.)

By means of this theorem we can at once construct the image of any straight line formed by reflexion at a plane surface. Let FG (fig. 5) be such a straight line, and let fg be the images of the two extreme points F and G, which will lie in the plane $FNNG$ drawn through FG perpendicular to the reflecting plane. And all the foci of reflected rays corresponding to points between F and G will lie between f and g; hence the image of FG will be a line fg, equal to it, but situated on the opposite side of the plane, in an inverted position, and geometrically symmetrical with FG. This imaginary line will always have the same position so long as FG is fixed; and in whatever way it be viewed, it will always produce the same impression as a real straight line would if substituted for it. For example, if we place an eye in the plane $FGNN$, which contains all the normals to the plane reflector, the two straight lines will appear to lie in that plane, as we know from the construction that they ought. But if we take the eye ever so little out of this precise direction, they will cease to be included in the same visual plane; so long at least as FG is not itself perpendicular to the reflecting plane, in which case the coincidence spoken of will take place in all directions around FG. In this case the imaginary straight line fg will always appear situated on FG produced, but in an inverted position. This affords then a practical test whereby to recognize, by means of reflexion, whether a straight line is perpendicular to a given polished plane. It is this test which will enable us to prove with the most extreme rigour, that the direction of the vertical is, at every place, normal to the surface of a still fluid.

25. To apply the test, take a large vessel VV (fig. 6), and fill it almost to the brim with a fluid which will reflect strongly, such as mercury or blackened water. The vessel is supposed to be of considerable size, because near the edges the fluid assumes a form determined by gravity, and by the action upon the fluid of the matter of which the vessel is composed. But this latter force, which is of the nature of chemical forces, diminishing in intensity very rapidly as the distance increases, its effects become altogether insensible at a small distance from the edge; and the remainder of the fluid will assume the form which is determined by the action of gravity only. It is then only this portion, namely, that which is unaffected by the action of the sides of the vessel, with which we shall be here concerned, and which we can make as large as we please by increasing the size of the vessel.

Moreover, it is easy to ascertain that this surface is sensibly plane. For if we look at the images of objects reflected by it, these images are exactly similar in form to the objects, and appear to be situated at an equal distance from the surface, and on the opposite side of it. These optical relations are those which belong exclusively to reflexion at a plane surface. As a particular case, the image of a straight line formed by reflexion at a plane surface is also a straight line similarly situated on the opposite side of the reflector, as has been shewn in the preceding article. Now we are able to observe, that this is very accurately the case for all portions of the reflecting surface in question, which are at some little distance from the sides of the vessel. This portion therefore is *sensibly* plane. Here, as in the case of the parallelism of the vertical lines, it is necessary to introduce this restriction; for the same appearances would be sensibly exhibited if the reflecting surface, instead of being *mathematically* plane, were only spherical, or even spheroidal, with a very large radius; and in matter of fact we shall soon see that this is the actual case before us.

Now suspend over this surface a plumb-line FG; and, to render the observation of the inverted image more easy, take as the weight a small homogeneous cone of metal, very carefully suspended, having its vertex turned towards the fluid, and almost in contact with it. If the image of the plumb-line be observed, it will be found that it has the appearance of the line itself reversed, and that it coincides with the production of the real plumb-line. The vertical therefore which this plumb-line represents is perpendicular to the reflecting plane surface.

To establish this result with greater rigour, suspend outside the vessel another plumb-line $F'G'$, and place the eye at O in such a position that the line $F'G'$ may exactly eclipse the line FG. Then the image fg will be found to be also eclipsed. But if the eye be moved ever so little, to the right or left, the image fg as well as FG will reappear; and both will follow the direction of $F'G'$ so exactly that they may be made apparently to touch it along their whole lengths. It is quite certain therefore that the image fg lies wholly in the vertical plane which contains the two parallel lines FG, $F'G'$.

This test can be applied on all sides of FG, and it will always be with the same result. That is to say, if we suspend a third plumb-line $F''G''$, the image fg will be found to lie in the plane which contains FG and $F''G''$. Moreover the several *foci* of reflexion of which this image is composed are situated with respect to the surface similarly to the corresponding points in FG, but on the opposite sides of the surface. In this case, then, the locus of such foci lies at the same time in two vertical planes drawn through FG, which cannot be unless it actually coincides with FG produced, as the observed appearance in fact indicated that it did. And such a coincidence cannot take place except in the single case in which all the perpendiculars drawn from different points of FG to the reflecting plane, coincide with the direction of FG produced. This relation therefore must hold between the vertical and the fluid surface, which is in conformity with the proposition required to be established; namely, that *the surface of a fluid at rest, which is sensibly plane, is every where perpendicular to the vertical direction, as determined by the plumb-line.*

26. It is easy to understand how this relation of perpendicularity contributes to the equilibrium of fluid masses contained in vessels. The material particles which compose these masses are subject to the action of gravity, as are those of all other material bodies. Whatever may be the direction in which gravity may act, it is certain that it acts upon all the particles in that same direction. Thus, when we see the whole mass in equilibrium, we may at once conclude that the particles are disposed in such a manner that they can no longer yield to the action of gravity, being supported by the rigid sides of the containing vessel, and by their own mutual resistance. Equilibrium being thus established, suppose that the fluid mass is terminated in certain parts by a free surface, the form of which is influenced by no other force beside gravity; it will be necessary that this surface should be everywhere perpendicular to the direction of gravity; for, if this condition be satisfied, this force will only tend to press the particles of the surface towards the interior of the fluid, an effect which is counteracted by the incompressibility of the fluid; whereas, if the surface were oblique to the direction of gravity, gravity would tend to make the particles move, without anything to counteract the tendency; and thus the fluid could not remain at rest with a free surface so formed.

This condition of perpendicularity subsists, and is still sufficient for the equilibrium of the surface, when it is pressed at every point by a heavy fluid specifically lighter than itself, as for example, air pressing on the surface of water. For, an equal pressure being exerted at every point perpendicularly to the surface of contact, the direction of this pressure must coincide with the direction of gravity if the fluid surface be perpendicular to the direction of this latter force; and the incompressibility of the fluid, combined with the resistance of the rigid sides of the vessel, will then be able to support the particles of the free surface under the action of these two forces.

27. But how can a fluid mass, composed of heavy particles, be put and maintained in equilibrium, if, instead of being contained in a vessel, it be free in space and separated from all kind of support, as we have ascertained to be the case for the earth and the waters upon it? It still may be, provided that the form be such as to be every where perpendicular to the direction of gravity, and that the forces which tend to press each particle normally into the mass be equal to the interior reaction which tends to push the particle outwards. And since the earth, composed as it is of land and water, does thus exist isolated in space without the fluid particles at its surface indicating any tendency to move laterally, it follows that the exterior figure of the fluid mass must be determined by the above condition; and since we find that the actual figure is spheroidal, the absolute direction of gravity must vary from point to point on its surface, being always normal to the surface as represented in fig. 7. Moreover, this direction, which we have called the vertical, is indicated at each place by the plumb-line; it follows then, that these plumb-lines, and the verticals which correspond to them, must be inclined to each other at different points of the surface, and must be diametrically opposed at opposite points; and that gravity which gives them their direction must act upon heavy bodies in the direction perpendicular to this surface, or retain them at the surface by help of the resistance which the surface offers to their penetration.

28. In consequence of this spheroidal form, when we rise above the earth's surface, as upon a mountain, or in a balloon, we have a more extended view the higher we rise; and the cone of visual rays, which limits the extent by its contact with the surface, are inclined more and more to the vertical of the observer, as shewn in fig. 8. The curve of contact thus formed always appears sensibly circular round the observer; which confirms all the other indications we have found of the general form of the earth being, if not exactly spherical, at least that of a spheroid, the radius of which is very great, compared with the irregularities existing

on its surface, although these irregularities appear enormous when we compare them with our own dimensions*.

28. The extent of surface visible varying thus with the altitude of the place of observation, it would be reduced to a mathematical point if we could imagine an observer whose eye was situated at the very surface of the sea. To preserve exactness of expression, it is agreed, in Astronomy, to denote by the term *horizon*, a plane drawn through the eye of the observer, perpendicular to the vertical. This plane is supposed of indefinite extent in all directions. In fig. 8, HOh represents the horizon, and the angle HOH', which is the angle made with this plane by the lowest visual ray, is called the *dip of the apparent horizon.* This angle is always much smaller than it is represented in the figure, in which we are obliged, in order to render it sensible, to exaggerate the dimensions of the mountain relatively to the size of the earth†.

29. Generalizing these terms, we call every plane drawn perpendicular to the vertical at a given place, a *horizontal plane*, and every line traced in such a plane, a *horizontal line.* It is evident that such a line is sensibly perpendicular to all verticals drawn through different points of it, for a small space around the primitive vertical.

This characteristic enables us to trace horizontal lines, and to recognize their horizontality, by a very simple and common proceeding. Construct a metal plate, so thick as not to bend, and give it the form of a triangle ABC, (fig. 9), having the sides CA, CB exactly equal. Mark D the middle point of AB, and trace a fine line CD, which will of course be perpendicular to AB. In CD pierce two fine holes P, P'; and between P' and D make a considerable aperture OO.

* The mountain of Mongo, in Spain, on the shore of the Mediterranean, which is 2385 feet in height, appears from the sea as a small island at a distance of 20 leagues; and, conversely, from the summit of Mongo, we may see, at 20 leagues distance, the islands of Iviza and Formentera, which from the shore are not visible. In Mexico, the perpetually snow-clad summit of Orizava, which is 17405 feet in height, may be seen, according to Humboldt, at a distance of 60 leagues. An observer on this summit would therefore see his horizon extending 60 leagues about him.

† Supposing the height of the mountain to be equal to that of Chimborazo, or 21415 feet, and representing it as in the figure, it would be necessary to give the earth a radius of nearly 7 feet.

Now suspend a fine plumb-line by passing one extremity through P and knotting it, and allow it to hang in such a manner that the weight G may appear in the aperture OO. This apparatus will be nearly like an ordinary mason's *level.* Hold it in such a position that the plumb-line, hanging freely, may coincide with the line PP'; a coincidence which may be ascertained either by its striking against that line, or oscillating equally on the two sides of it. This coincidence being established, the line PP' will be vertical. Consequently, AB which is perpendicular to it will be horizontal; and if a long thread HH be stretched in such a way as to coincide in direction with it, this thread will also be horizontal throughout its whole length, at least if it does not bend sensibly from its own weight.

The determination thus obtained will be the more exact according as the plane ABC is larger, the sides CA, CB more exactly equal, the straight line PP' finer, and more accurately perpendicular to the base AB at its middle point. Whatever pains we take, the results will be very far from possessing that degree of exactness which Astronomy requires, and to which it is actually able to attain; but they will furnish a first approximation which can be rendered more and more exact by methods more and more refined, until we arrive at an almost ideal degree of rigour, such as at present can scarcely be conceived.

If we suppose the plane ABC, and the horizontal line HH, to turn simultaneously round the plumb-line PG, this line coinciding constantly with PP', HH will trace out a plane perpendicular to the vertical PG and consequently horizontal with reference to the point D. Conversely, if we construct round D a plane such that when we place AB upon it in all directions, or in two directions, for that will suffice, the plumb-line PG rests upon the straight line PP', the plane will be horizontal. This is the proof which is practically employed in building to make a surface plane and *level.*

Fig. 10 represents another method, equally simple, of tracing a horizontal line, by means of a *water-level.* It is composed of a metallic tube TT, about a yard in length, and having an internal diameter of one or two inches. At the extremities TT, are two glass tubes, of the same diameter, at right angles to the former, so that the whole forms a continuous canal with two rectangular branches. Let the canal be nearly filled with water, not entirely, but in such a manner that *TT* being nearly horizontal, the free surface of the water may appear as at *EE* without overflowing. When after some oscillation the fluid comes to rest, the straight line *EE* touching the two surfaces is horizontal. Then following the line so determined by a visual ray produced on both sides, we can determine two points *HH*, which lie upon it; and joining these points by a straight line, this will be horizontal also.

CHAPTER IV.

ON THE QUANTITIES EMPLOYED IN ASTRONOMY TO DETERMINE THE APPARENT POSITION OF A HEAVENLY BODY. DEFINITION OF THE MERIDIAN LINE.

30. We have now established by sure proofs, that the general system of terrestrial land and water constitute a body of spheroidal form, all the particles of which are held together by gravity which attracts each of them towards the entire mass. We have ascertained besides that this spheroid exists isolated in space, at a great distance from all the heavenly bodies; which *seem* to revolve continually round it by a simultaneous and general motion, the phases of which are gone through in the course of a day and a night.

The expression of doubt which has just now been employed in speaking of this last phenomenon, may excite some surprise. For the revolving motion of the heavens presents itself to us as an evident fact. But when we examine the mechanical conditions under which such motion could take place, we very soon begin to doubt whether it can be real.

For, this simultaneous revolution of all the heavenly bodies is accomplished and repeated continually, without any change in the relative positions of the bodies, which are at the present time such as they were described by Ptolemy seventeen centuries back. This constancy of relative position requires a physical cause which can maintain and supply a mutual bond of connexion amongst the heavenly bodies. But their occultations have given us proof that they are placed at different distances from the earth; the moon not being so remote as the sun and planets; the planets again less remote than the fixed stars; and these last at an enormous distance, such that the power of our best telescopes represents them to us only as mere points without appreciable dimensions. Nevertheless these points, which possibly are so many suns, are also at unequal distances from us. For when we observe them with telescopes, we see some of them eclipse each other, and then separate, in consequence of relative displacements, which must be very considerable, although to us they are scarcely perceptible, on account of the great distance from which we view them*.

Now since all these follow each other during each diurnal revolution of the heavens, always offering to our view the same apparent relative position, it is necessary, if this motion be real, that each of them should have an absolute velocity in its diurnal course, exactly proportional to its distance from the earth; and, by consequence, it will be necessary that the terrestrial spheroid should exert some power, proper to itself, capable of producing, graduating, and maintaining, the exact proportion of this motion, amongst all the millions of stars which thus revolve continually round the earth throughout all the remote parts of the heavens.

Though the notion of such a power does not present, strictly speaking, any absolute mechanical impossibility, it is, to say the least, one of an extremely complicated kind; and the result is contrary to all the analogies which the whole of the other observable physical motions present to us. For we have no other instance of a single body, or a single material particle, governing in the manner supposed an infinity of other particles by its sole power, and impressing upon them a common motion of revolution about itself, without the manifestation amongst the particles so influenced of any action depending upon their mutual distances; so that they should appear inert and insensible to the action of every other force, except the central power of the particle which governs their motions. These two characteristics therefore, of a kind of action absolutely without any other examples, and, at the same time, exerted under conditions of extreme complication, attach a very great amount of physical improbability to the explanation of phenomena which requires them; and though such explanation may appear to have on its side the evidence of the senses, yet, before we can consent to admit it, it is necessary to inquire whether the same observed results may not be

* This has been observed for the following stars, *Atlas* of the constellation Pleiades; η Herculis; ω Leonis; γ Coronæ; γ Virginis. All these stars, viewed through very powerful telescopes, sometimes appear *double*, that is to say, composed of two distinct stars, separated by a very small visual angle. The two components of these binary systems, are observed to approach, to coincide, then again to separate.

produced by some other mechanical combination, free from such extraordinary conditions.

And there is in fact an hypothesis which will account for all the appearances presented, with admirable simplicity. It is this, that *the diurnal revolution of the heavens from east to west is only an optical phenomenon, caused by an actual rotation of the earth in the opposite direction, that is, from west to east.*

In the first place, with regard to the preservation of the same appearances, this is evidently quite complete according to this method of conceiving them; and even if we discuss them logically, there is no one test to indicate whether it is the heavens or the earth which revolves. The mind seizes upon the former of these suppositions first, merely in consequence of our being accustomed all our lives to witness the motions of terrestrial bodies taking place round about us, as though the earth were fixed; and if it be the earth itself which is turning, they will turn with it without their relative displacements receiving any sensible modification in the general observations which we make. But on this hypothesis all the complication of the diurnal motion of the stars disappears, as well as the physical difficulties which accompany it, since they become altogether extraneous to this apparent revolution, and have no other motions than those which may individually belong to them. As to the possibility of the terrestrial spheroid thus turning constantly about an axis in itself, together with all the bodies fastened to its surface by gravity, far from being subject to mechanical difficulties, this is, on the contrary, entirely conformable to the general laws of motion. For, in order that a material body, isolated in space, should not thus revolve, it is necessary that it should never be acted upon by any extraneous force, or that if it be, then the direction of the resultant of the forces which act upon it should always accurately pass through its centre of gravity. So that the immobility of such a body, and its non-rotation, would be circumstances altogether exceptional. Moreover, we can ascertain, by help of the telescope, that all the heavenly bodies which have a sensible disk do actually revolve, with different angular velocities; for example, the sun in twenty-five days and a half, the moon in twenty-seven days and a third, Venus in rather less than a day (0.973^d). It is then at once mechanically simple, and physically in accordance with analogy, that the terrestrial spheroid should have, like the other heavenly bodies, a motion of rotation, which being completed in a day and a night will cause us to see all the other heavenly bodies as though they were revolving about the earth in the opposite direction, in the same interval of time.

Simple and natural as this conception appears, it required thousands of years of observation and thought to arrive at it. Those of the old philosophers who first proposed it were looked upon with distrust; and the immortal Copernicus, who revived it in Europe in the sixteenth century, and developed all the analogies which supported it, was ridiculed on the stage as a silly dreamer. In the present day it has become a common and unquestioned notion. Nevertheless, in this book, which is intended to teach philosophically, we shall not at present admit it as such, but only as offering a representation of observed facts, more simple and physically more probable than the supposition of a real motion of the heavens. Indeed, in the geometrical constructions which we are about to introduce, for the purpose of fixing the relative directions of visual rays drawn to the different parts of the heavens, we shall continue to consider the earth's surface as an immovable foundation for our instruments; it being understood that we shall restore to them the actual motion of rotation which is communicated to them, when we wish to discuss the indications which they have supplied, in order to conclude from them the actual motions of the bodies observed.

31. If the earth have an actual rotation, the diurnal revolution of the heavens must appear to take place round an imaginary rectilinear axis passing through the terrestrial mass. This apparent motion would seem exactly circular to an observer situated at any point on the axis of rotation; and it would appear uniform or variable according as the earth itself is revolving at a uniform or variable rate. These are points which cannot be settled *à priori*, but must be deduced from exact observation; and it is so much the more necessary to have them settled, because they will enter as necessary elements into all the observations which we make of the positions of the heavenly bodies, as also into all the conclusions which we may draw concerning the actual nature of their motions. We proceed then experimentally to this funda-

mental determination, devising from step to step the necessary means of making the required observations; and which, from being in the first instance of a very simple kind, will soon arrive at that almost ideal precision which constitutes the power of astronomy.

32. In a situation where the view is not confined, and from whence, if possible, we can observe the sun rise and set upon a sea-bounded horizon, choose a space already nearly horizontal, and construct a platform or floor of hewn stone, well cemented; and level the upper surface with the greatest care possible, according to the methods described in the preceding Chapter. Then procure a large slab of stone or marble, having one of its faces exactly plane; near the edges of which cut a continuous groove, which shall be exactly level with the wrought surface, as represented in fig. 11. You may determine, with a considerable degree of accuracy, the plane state of this surface, by trying whether a well-squared piece of metal will coincide with it exactly in all directions, and also by examining whether the reflected images of very fine straight lines appear rectilinear, like the lines themselves. Next, place this slab upon the platform of masonry, in such a manner that water or mercury poured into the groove may fill it completely everywhere, without running over at any part. This condition being fulfilled, fix the slab permanently, without deranging its position. Its surface will evidently be horizontal, since it will coincide with the form which the fluid at rest in the groove would assume, if we increased it sufficiently to occupy the whole surface of the slab. This arrangement was in fact employed in China, towards the middle of the thirteenth century, by the Arabian astronomers attached to the court of the Emperor Cobylai; and the first exact measures of the sun's shadow, of which we shall have to speak presently, were made by the great Chinese astronomer, Cocheouking, at about the same period, upon large squared surfaces of plane slabs, the level of which he ascertained by the preceding process*.

* The use of water for the purpose of rendering a plane horizontal was not unknown to the Greek astronomers, from whom the Arabians may have borrowed it. Theon, in his commentary on the *Almagest*, describing one of Ptolemy's instruments, says that it should be fixed upon a plane rendered

Now take a thick rectangular piece of metal having its surfaces made carefully plane, forming with each other accurate right angles, the exactness of which may be tested by the reflexion of light. Through H H' (fig. 12), the middle points of its extreme edges DE, $D'E'$, trace a very fine straight line, which will of course be parallel to the longitudinal faces. Then apply and fasten to the terminal faces two plates of metal, also plane and rectangular, the edges of which are erected perpendicularly to its length. Each plate must have an aperture cut in the midst of it to allow the light to pass through. This being done, from the points H H' stretch in these openings two threads HF, $H'F'$, reaching to the middle points of the opposite sides AB, $A'B'$. If the arrangements have been made carefully, the two threads FH, $F'H'$ will be parallel to each other, and perpendicular to the straight line HH', and they will be found to be in the same visual plane with that line, a point which may be immediately verified. Then if the piece of metal be very regular in its thickness, when we place its lower face upon a horizontal plane, such as $ENWS$ (fig. 11), the line HH' will be horizontal also, and the two threads HF, $H'F'$ will be vertical, which may be also verified by comparing them with a plumb-line. We shall thus have an apparatus resembling that used by surveyors, and which, for brevity's sake, I shall call a *sight*. In order that the indications of this instrument may have some degree of precision, we must suppose its base to be two or three yards in length. But the horizontal slab (fig. 11) upon which we place it may be of smaller dimensions, if this latter be fixed upon a solid mass of isolated masonry, which rises a little above the surface of the ground, and is slightly overlapped by the circumference of the slab.

horizontal either by means of the plumb-line, or by pouring water upon it and raising the plane by means of small wedges until the water remains upon it at rest. But Ptolemy, when he speaks of the same instrument in his *Almagest*, does not indicate this excellent method of adjustment. And in general by the want of a spirit of precision which he manifests in the description of methods of observation, one is inclined to believe that he was only a good mathematician and laborious calculator, but an indifferent observer. We have so much the more reason to regret the loss of the original works of Hipparchus, which Ptolemy only cites by fragments far too scanty, and of which the *Almagest*, having at that epoch become more popular, has probably deprived us for ever.

These arrangements being made, take advantage of the latter part of a fine night; and towards morning, before the sun rises, place the *sight* upon its plane (fig. 11), and direct it towards that part of the heavens in which the disk of that body is about to appear. Then, placing the eye at a little distance from one of the threads in the plane which contains them both, turn the instrument gradually, so that this plane may contain the first point of the disk which makes its appearance. Trace upon the plane the line $O\sigma$, coinciding with one of the edges of the base of the instrument. This will be the trace of the *vertical plane* in which this first point shews itself, and if through the centre C of the plane we draw a straight line $C\sigma$ parallel to $O\sigma$, $C\sigma$ will be the trace of the analogous vertical, supposing that the observation had been made from the point C; for the extent of the slab is inappreciable compared with the distance of the sun. To render this determination more exact, repeat the same observation upon the point of the disk which last quits the horizon; and if the new straight line $O\sigma$, when transferred to $C\sigma$, differ ever so little from the line determined by the first observation, take the line lying halfway between them as the true position of $C\sigma$. This will be the horizontal trace of the vertical plane in which the sun's centre rises.

On the evening of the same day, repeat the same observations towards the west, when the sun sets; and we shall thus have another line $C\sigma'$, which will be the horizontal trace of the vertical plane in which lies the centre of the disk when it disappears below the western horizon. Divide the angle $\sigma C\sigma'$ into two equal parts, and draw the bisecting line NCS.

We can determine this line by means of another apparatus, more simple, but less delicate, which is represented by fig. 13. CG is a cylinder of metal, of very small radius, turned exactly upon the production of a larger cylinder HH' which serves for a base when it is placed upon the levelled slab of fig. 11. If these conditions of construction be accurately fulfilled, the cylinder CG will be vertical; and we can assure ourselves that it is so by observing whether it corresponds accurately with a plumb-line, when we turn it round upon its base. This being supposed, we must wait until the sun appears in the morning in the plane of the levelled slab, and we must trace upon the slab the axis of the shadow cast by the vertical cylinder CG; this will be the horizontal trace of the vertical in which the sun's centre will then be found. We must repeat the same operation on the evening of the same day; and by bisecting the angle contained between the two shadows we shall have the line NCS.

Let us suppose that the preceding observations have been made, at some place in Europe, about the 22nd of December. This is the time of year when the daily arc described by the sun is least elevated above our horizon; and at this limit of its proper motion, at which after having ceased to descend it begins again to rise daily towards our zenith, its diurnal change in this direction is almost imperceptible for many days, so that its diurnal arc is almost identical with that which a fixed star, situated at the same part of the heavens, would describe. This phenomenon is repeated for a like reason about the 22nd of June, when the sun having attained to its nearest distance to our zenith, commences describing less elevated diurnal arcs. These two limits of its annual motion are called the *solstices*, to express the constancy of diurnal elevation which we observe at those times. Moreover, if at these two epochs, we determine the directions of its rising and setting, in the manner just now explained, they will evidently be very different. Nevertheless, in the two cases the same straight line NCS will bisect the angle between them exactly.

This result would not be obtained, at least not with so much strictness, if the observations were made at the intermediate phases of the sun's motion, in which the diurnal arc described by the sun undergoes a sensible progressive displacement. Supposing, for instance, that it is rising higher and higher, day by day, towards our zenith, if the sun rises to-day at σ (fig. 14), and sets at σ', this last position will not result merely from the diurnal motion of the heavens, but from this motion combined with the transfer of the sun towards the zenith, whilst it has remained above the horizon. We can, however, easily correct the effect due to this latter cause; for after having observed the sun in the morning at σ, and in the evening at σ', we shall only have to observe it the next morning at its rising, which will be at a point such as σ'' in consequence of its proper motion. Then if the angle $\sigma C \sigma''$ be appreciable, we must divide it by a line CX in such a manner that σCX shall be to $\sigma C \sigma''$ as the duration of the visible day at that period of the year is to the total duration of a day and night. The line CX will mark very approximately the direction at which the sun would have risen to set in the direction $C\sigma'$, if there had been no proper motion towards the zenith whilst it remained above the horizon; and the angle $XC\sigma'$ thus obtained, will be found to be accurately bisected by the line NCS, determined at the times of the solstices.

The preceding correction, destroying as it does the effect of the proper motion of the sun towards the zenith, reduces it in reality to the condition of a star, which would have no such proper motion. We ought, then, to expect that the same bisection will be made by the line NCS if the angle $XC\sigma'$ were given by the rising and setting of any star. And this is the fact, as may be easily proved by choosing a star sufficiently bright to be capable of having this method of observation applied to it in the course of one and the same night.

33. These results may be realized at all parts of the earth; there always exists, for each point of the earth's surface, a horizontal straight line which thus bisects the horizontal directions of the rising and setting of all the visible heavenly bodies; and this characteristic property may serve to define the line. It is called the *meridian line*, or simply the *meridian of the point of observation*. We shall find, afterwards, means infinitely more precise for determining its direction upon the horizontal plane. But the property by which it has been here defined will be found always to belong to it exclusively.

The meridian being produced indefinitely in the plane of the horizon which contains it, determines in the heavens two opposite points, which are the true *north* and *south*. For Europe, the south point is situated in that part of the heavens at which the sun attains each day to the highest point of its diurnal arc. The north point, opposite to the preceding, is found in that part of the heavens where may be seen those seven stars which compose the constellation so well known as the Great Bear, or as Charles' Wain.

If through the centre C of observation (fig. 11), we conceive another straight line EW to be drawn in the plane of the horizon and perpendicular to the meridian, this line produced indefinitely will meet the heavens in two points, which are the true *east* and *west* points. The four points, east, west, north, and south, are known collectively as the *cardinal points.*

Lastly, the plane drawn through the meridian line and the vertical is called the *plane of the meridian,* or simply, the *meridian.* The plane drawn through the vertical perpendicular to the meridian is called the *prime vertical.*

34. We shall now be able to explain the manner in which the position of a body in the heavens is defined. Let $C\sigma$ (fig. 15), be the direction of a visual rectilinear ray, drawn from C to the body σ at any given instant. Through the ray $C\sigma$ and the vertical CZ conceive a plane to be drawn of which CV is the horizontal trace. This will be the *vertical* of the heavenly body (§ 23); and its direction will evidently be defined if we know the angle VCN formed by its trace with the meridian line, reckoning from the north point. This angle is called the *azimuth of the vertical,* or the *azimuth of the body,* a name derived from the Arabians; and its value is reckoned continuously from 0^0 to 360^0 beginning from the north point. It remains only to determine the position of the ray $C\sigma$ in the plane so defined. And this position will be fixed, if we know the angle σCV which is formed by the ray with the horizontal trace of the vertical, and which we call the *apparent altitude* of the body; or it will be the same thing if we know the angle σCZ formed by the ray with the vertical, reckoned from the zenith, an angle which is called the *apparent zenith distance.* This is evidently equal to the complement of the altitude.

35. To obtain these angles we *observe* them with instruments supplied with metallic circles graduated upon their circumferences, and of which the general type is represented in its simplest geometrical form in fig. 16. $ESWN$ is one of the circles, which is to retain a horizontal position and measure azimuths. It is called the *azimuthal circle.* At its centre rises an imaginary rectilinear axis $CC'Z$ which is perpendicular to its plane, and which consequently remains vertical. A

second circle is fixed upon this axis by one of its diameters, so that its plane is vertical; and it can thus turn round CC' into the plane of any vertical, on which account it is called the *vertical circle.* An imaginary straight line $C'\sigma$, a radius of this circle, can assume all possible positions in its plane, around the centre C'; it is called the *optical axis.* When we direct it towards a heavenly body σ', the angle $\sigma' CZ$ which is the zenith distance, may be *read off* upon the graduated arc $Z\sigma$. At the same time, the horizontal trace of the vertical circle is marked upon the azimuthal circle by a rectilinear arm CV, moveable round the centre C and which the vertical circle when turned azimuthally carries with it; so that the azimuth NCV' is read upon the horizontal arc NEV, upon which has been previously marked the direction of the meridian NCS which passes through its centre. The two required angles are thus determined simultaneously.

This is only an abstract description, expressing the geometrical conditions which actual instruments ought to realize. But in these, the straight lines CV, $C'\sigma$ are replaced by bars of metal and by telescopes, of sensible dimensions, in which case it is necessary to find the imaginary directions of the lines which no longer pass through the centres C and C'. Moreover, the axis CC' can no longer be regarded as a mathematical line; it results from a rotation round a material axis, the exact position of which must be discovered and its verticality determined, as also we must ascertain whether the imaginary point Z coincides with the zenith point of the vertical circle. Finally, it is necessary to determine the mathematical position of the line NCS which passes through the centre of the material azimuthal circle. All these determinations require special experiments which constitute what is called the *adjustment of the instruments;* and they should be effected by methods adapted to the construction of the various instruments, which are not always intended to give both the position-angles of a heavenly body, but only one of them, either the azimuth, or the zenith distance. It will be necessary therefore, to follow this course, and explain individually the adjustments which each particular instrument will require when we begin to make use of it. But the conditions necessary to be fulfilled are always particular cases of the preceding general explanation, when it is limited to those elements of observation which each particular instrument is intended to supply.

36. Without anticipating these researches, we shall here explain a very simple method of observation, which can in truth only serve in the case of the sun, and which the perfection of modern optical instruments has quite thrown out of use. But it ought not to be passed over in silence, because for many centuries it was the only known method; and moreover, in the hands of skilful observers it has furnished observations which have become, on account of their antiquity, of great value to Astronomical science. This will be the object of the following chapter.

CHAPTER V.

ON THE GNOMON AND ITS USE IN DETERMINING THE GENERAL LAWS OF THE SUN'S PROPER MOTION.

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37. To give this apparatus a degree of precision which shall enable us to deduce from it Astronomical results, it is necessary to be able to adapt it to some large vaulted building, the walls of which may be considered as not liable to any shaking movement. The floor within must be *levelled;* then paved with well-joined flat slabs, the common surface of which is made exactly horizontal throughout its whole extent. To fix the mind by a particular example, suppose this construction made for our temperate climate of Europe, where the sun in its diurnal course always crosses the meridian above the plane of the horizon, and to the south of the zenith, a point which it never reaches. At the spring of the vault, on the south side, make an aperture and insert in it a circular plate of blackened metal, having a very fine hole pierced in its centre; and let this plate be arranged so as to be perpendicular to the meridian, and oblique to the vertical, in such a manner that its southern face may be always exposed to the solar rays when the sun crosses the meridian, or is nearly in that plane. These arrangements being made, fix the plate permanently, and let no light be allowed to enter, except through the small central hole in the plate, which is represented by C in fig. 17.

From C let fall a plumb-line, which by its intersection G with the floor will shew the foot of the vertical which passes through the centre C of the small hole; measure the height CG with as much care as possible. Through the point G trace the meridian line NS, as well as the line EW perpendicular to it, which may be done either by the help of previous determinations, or by the indications which the instrument itself furnishes, as we shall see presently. When the sun is above the horizon, to the south of the plate, its light, transmitted through the hole C, casts on the paved floor a luminous image of elliptic form, which the shadow of the other part of the plate and of the walls surrounds, and which by this contrast becomes perfectly distinct. So that if at any moment we trace upon the floor the circumference of this image, we have the horizontal section of the luminous cone which at this particular instant is passing through the aperture *C*.

The apparatus thus arranged is called a *gnomon;* it was employed for the first time, with the careful construction just described, by the skilful Chinese Astronomer Cocheouking, in the year 1271. The vertical height *CG* was 40 Chinese feet, or rather more than that number of English; the aperture *C* was as fine as a needle. Almost four centuries elapsed before the astronomers of Europe had an instrument comparable with this for magnitude and for precision. That alluded to was constructed, in 1653, by Dominic Cassini, at Bologna, in the church of St Petronius. The height *CG* was about 83 feet, and the diameter of the aperture an inch. It was pierced in a thick plate of bronze, inserted into the vault of the building, as has been supposed in the preceding description.

In order to appreciate the indications of such an apparatus, it is necessary first to analyze the formation of the luminous image traced upon the floor. This may be done by the help of fig. 18, which represents the profile of the luminous cone transmitted through *C*. Each point of the solar disk, such as σ', σ'', transmits through the aperture a fine cone of light, which, in consequence of the sun's distance, may be regarded as an oblique cylinder having the aperture for its base; and the section of this cylinder by a horizontal plane is a small luminous ellipse, the centre of which, R', R'', may be supposed to belong to the ray which passes through the centre of the aperture, with a smaller amount of error in proportion as the aperture is finer. All the cylinders, which emanate from the different points of the disk, will give in like manner so many small images which will overlap one another, as represented in fig. 19; and the centre of the figure so formed will differ very little from the point Σ (fig. 18), in which the ray $\sigma C \Sigma$, drawn through the centre of the sun's disk and the centre of the aperture would meet the floor. If then, we measure upon the floor the distance ΣG, as we have measured CG, we shall know two sides of the right-angled triangle ΣGC. Thus we can calculate the angle at C, which is opposite and equal to σCZ, that is, to the zenith distance of the centre of the sun's disk for the instant at which the point Σ was marked on the floor.

In applying this to the image Σ of fig. 17, or rather to its central point, we see that the plane ΣGC drawn through the centre of this image, is the vertical plane in which lies the centre of the disk at the moment in question. The *shadow-line* ΣG, produced in the direction GV, to the south of the gnomon, marks the horizontal trace of this vertical plane; and the angle NGV is the azimuth of the centre of the disk, reckoned from the north point towards the east. The angle ΣGN is evidently the supplement of this azimuth. To obtain its value, we must draw $\Sigma\Pi$ perpendicular to the meridian, and measure its length. Then in the right-angled triangle $\Sigma G\Pi$, we shall know the hypothenuse ΣG, and the side $\Sigma\Pi$ opposite to the angle at G. This angle, therefore, can be calculated, and its supplement will be the azimuth of the sun's centre. The two position-angles of the sun's centre will thus be determined.

38. According to the arrangement which we have given to our apparatus, when the sun shines upon the southern face of the plate before passing the meridian, the luminous image Σ will fall to the west of the meridian line. It will fall upon the meridian line when the body is itself upon the meridian; and it will pass to the eastern side when the sun has crossed the meridian. If we measure its distance from the point G, or the line $G\Sigma$, in these various phases, we shall find that for the same day it is shortest when it coincides with the meridian line; which gives the angle at C in the variable triangle ΣCG its smallest value. But this angle is always equal and opposite to the sun's zenith distance. Consequently the sun has each day its least zenith distance when it crosses the meridian; and, conversely, its apparent altitude above the horizon is then greatest.

39. Let us then first employ ourselves upon this remarkable phase; and, for that purpose, let us measure each day the length ΣG of the meridian shadow, during the whole of

a solar year, comprising a complete revolution of the seasons*. We shall find that it attains its maximum value about the 22nd of December; and then its variations, from one noon to another, are almost insensible. The sun therefore describes at that period its lowest arcs above our horizon; and as it appears stationary in this depressed position for some days, this epoch is called the *winter solstice.* But very soon afterwards the length of the meridian shadow begins to decrease daily; which shews that the solar diurnal arc is rising. It attains its least value about the 22nd of June; and then again it varies very little from one noon to another. The sun then describes its diurnal arcs nearest to the zenith; and it appears to be stationary for some days, on which account this epoch is called the *summer solstice.* But very soon the sun begins to descend from this elevation, and approach the south. The length of the meridian shadow increases every day, until at length it again reaches its maximum on the 22nd of December. After which it goes through the same cycle of changes.

40. It is natural to inquire how many meridian transits of the sun this period embraces. For this purpose, suppose that on a certain day we have found the value of ΣG (fig. 20) corresponding to the meridian shadow, and that this observation is made at a season of the year when the sun is daily approaching the zenith. The next day, the meridian shadow will be shorter; and it will decrease until the summer solstice. Then it will again increase daily, will pass the point Σ, and continue to lengthen until the winter solstice. After arriving at this limit, it will again begin to decrease, which will bring the meridian image continually nearer to the point Σ, where we first observed it, in the corresponding phase of its variation. But the return to the length ΣG will be found not to take place after an exact number of meridian transits. For three hundred and sixty-five transits will bring the meridian image to σ_1, where it will not have reached the point Σ_1; and

* The word *year* is here taken in its vulgar acceptation, without attaching to it the notion of a precisely defined interval of time. It is science alone, which in the process of its formation attaches to ordinary terms a fixed definite signification. But there is nothing to prevent us from employing them, in the first instance, with all the vagueness of their vulgar acceptation, in order to give the first description of facts.

three hundred and sixty-six will bring it to σ_2, where it will have passed Σ, and become nearer to G. Also we find that the interval $\sigma_1\Sigma$ is nearly a quarter of the whole daily variation $\sigma_1\sigma_2$. Then if we choose to denote by a *solar day* the interval of time which elapses between two consecutive transits of the sun across the meridian, the period of the motions of this body in altitude, that is, the time required for this motion to go through all its phases, will contain about three hundred and sixty-five days and a quarter, or $365^d.25$. This period is called the *tropical year*, because it expresses the entire completion of the sun's motion in ascending and descending towards the zenith; a term formed from the Greek word τροπη, which signifies a *turning* or *change of direction.*

The preceding determination may be materially confirmed by continuing to observe the return of the meridian image towards the point Σ, after four complete periods of its motion; for the first, which comprehends three hundred and sixty-five transits, having brought the image to σ_1, at a certain distance beyond Σ, the following period composed of the same number will place it still further from that point, the third further still; at length, after the fourth, the distance will be sensibly equal to the motion of the image in the course of one day, which will produce a coincidence at the following transit, which will be the three hundred and sixty-sixth. We shall thus have four complete periods in four times 365 + 1 days, or 1461 days; the fourth part of which will give us for a single period $365^d.25$; at least, very approximately.

41. We should obtain the same result by observing the returns of the sunrise to the same point of the horizon, with the *azimuthal* apparatus explained in § 32, and represented in fig. 11. For suppose that on a certain day we have traced, fig. 14, the direction CX of the sun rising at that season of the year when it is moving towards the north. If we observe it again after 365 days, we shall find that it has not yet returned to X, and that it rises in some other direction, as $C\sigma$ suppose. But the next morning, that is to say, after a lapse of 366 days, the rising will take place to the north of the point X, in the direction $C\sigma''$. If the instrument which gives these directions be good enough to shew their differences distinctly, we shall find that the first angular distance σCX is about a quarter of

the diurnal distance $\sigma C \sigma''$; and we shall conclude, as before, that the complete period of the return of sunrise to the same azimuth is approximately $365^d.25$; which may be also confirmed by the coincidence of the direction of sunrise with CX after 1461 sunrises. This method is found in the Sanscrit books. But there is no document which shews when it was devised, nor whether the Hindus really made use of it. And, in general, we have not at present discovered in their astronomical treatises the description of a single observation which is authenticated as belonging to them, or even which is given as such.

42. If in our temperate climate of Europe, for which the apparatus has been supposed to be specially constructed, we mark the series of points $\Sigma \Sigma_1 \Sigma_2$, fig. 17, which are traced out in one day by the centre of the luminous image, we find that they lie very approximately upon the arc of an hyperbola of which the meridian line is the axis. But the hyperbola varies every day in position as well as in form. At and about the time of the winter solstice it is convex towards the point G; whereas at and about the time of the summer solstice it is concave. Between these two epochs it passes gradually from one condition to the other; so that, on a certain day intermediate to these two epochs, and answering nearly to the middle of the interval, it sensibly coincides with a straight line qq_1q_2, perpendicular to the meridian. Admitting this coincidence to be exact within the limits of accuracy attainable in tracing a straight line, we see that, on the day on which this happens, all the luminous rays which come from the centre of the sun's disk through the aperture C are comprised in the plane drawn through C, and the line qq_1q_2. Hence, *on that day*, the sun in its diurnal course sensibly follows this plane; and as all the dimensions of the gnomon are infinitely small compared with the distance of that body, we see that it must at that time rise and set on the production of the line qq_1q_2, or its parallel EW, that is to say, at the true east and west points of the horizon. This is, in fact, what may be observed very approximately with the azimuthal apparatus of fig. 11. It must be remarked nevertheless that this result, as well as that of the permanence of the sun's motion in the same plane during a whole day, cannot be absolutely rigorous, since the

proper motion of the sun in the direction of the meridian taking place then as always, must, while it is above the horizon, carry it, be it ever so little, from the point W, which is opposite to the point E at which we suppose it to rise. But this deviation takes place in one direction when the sun is rising towards the summer solstice, and in the other when it is sinking towards the winter solstice; so that, omitting the displacement due to its proper motion, and considering only the course which the diurnal revolution of the heavens would produce, that is, considering it as a fixed star, we may suppose its path accurately contained by the plane passing through C and qq_1q_2.

43. It is of importance to determine the position of the plane in which so remarkable a phase is exhibited. We already know the trace qq_1q_2. It remains therefore to determine its inclination to the horizon. We shall obtain this by measuring the length q_1G of the meridian shadow on the day upon which the phenomenon takes place; for then, in the right-angled triangle q_1CG, we shall be able to calculate the angle C which is the meridian zenith distance of the sun, and the complement of the required inclination. If we determine also by the same method the meridian zenith distance of the sun at the two solstices, and construct in the plane of the meridian, fig. 21, the three rays $C\sigma$, CQ, $C\sigma'$ drawn from the point C to the sun's centre at the three epochs in question, we shall find that the zenith distance QCZ, corresponding to the epoch which we are considering is sensibly an arithmetic mean between the solstitial zenith distances. The plane $ECWQ$, which effects this bisection, is called the *plane of the celestial equator;* and the two epochs of the year at which the sun is in the plane are called the *equinoxes,* because the time of the sun's presence above the horizon, which constitutes day, is then sensibly equal to the time of its absence below the horizon which constitutes night. This equality is observed for all points of the earth's surface, under the same condition of bisection. But the absolute zenith distances of the rays $C\sigma$, CQ, $C\sigma'$, are generally different for different places; and this diversity furnishes a characteristic distinction to specify their position with reference to north and south, as will be explained hereafter. With regard to the angle σCQ, or $\sigma' CQ$, which is

formed by the equator with the two solstitial rays, it is found to be sensibly the same for the whole earth, and its actual value is not far from $23^0\,28'$*. This angle is called the *obliquity of the ecliptic,* for a reason which will presently be explained.

44. We find, from Ptolemy's *Almagest* †, that in his time, and even in the time of Hipparchus, there existed at Alexandria, in Egypt, a large copper circle, $EQWQ'$, fig. 22, permanently fixed in the position of bisection which we have assigned to the plane $ECWQ$, and which served to determine the epoch of each equinox, by observing the instant at which the upper part Q of the circumference, exposed to the solar rays, threw its shadow exactly upon that of the lower and concave portion Q'. If on that day the sun had moved accurately in the plane $ECWQ$, the shadow of the circumference would thus have been thrown in the plane of the circle throughout the whole day. But this could not in reality happen during any considerable space of time, on account of the proper motion of the sun in the direction of the meridian, and the moment at which the coincidence took place gave the very moment of the equinox. Nevertheless, this method of determination involved several sources of error, as will be better seen hereafter; and one of them was irremediable. For it depended upon a physical cause, of which no account was then taken, and which was sufficient to make the instrument indicate on the same day two epochs for the same equinox; the sun appearing to reach the plane of the circle, then to quit it by

* In fig. 21, the meridian line SCN, and the perpendicular ECW, which pass through the point C are denoted by the same letters as those used in fig. 17 to denote the analogous lines drawn through G. This is done with a view to indicate that these lines, although distinct and only parallel in the two systems, may be regarded as coincident in respect of their celestial extremities, the dimensions of the gnomon becoming insensible, when seen from a distance such as that at which the heavenly bodies are placed.

† Ptolemy lived at Alexandria, about A.D. 130, in the reign of Antoninus II. Hipparchus lived in the second century before the Christian era, and therefore was about 300 years before Ptolemy. He was of Bithynia, and observed at Rhodes. There is some doubt whether he ever observed at Alexandria. It is presumed that the equinoctial circle spoken of in the text, was established in this city by Eratosthenes, as well as other instruments due to the liberality of Ptolemy Philadelphus, about 250 years before the Christian era.

sinking again towards the south, and lastly, to reach it again by rising anew towards the north.

45. The two equinoxes of each year have received denominations which have reference to the phases of temperature which accompany them. That which happens when the sun crosses the plane of the equator, going northward, is called the *vernal equinox;* the other, which takes place when the sun is going southward, is called the *autumnal equinox.* The first is observed about the 21st of March; the second about the 22nd of September.

The solar year is thus divided by the two solstices and the two equinoxes into four parts, called collectively the four *seasons,* which have their names and limits fixed as follows:

The interval from the vernal equinox to the summer solstice is called *Spring.*

The interval from the summer solstice to the autumnal equinox, *Summer.*

The interval from the autumnal equinox to the winter solstice, *Autumn.*

The interval from the winter solstice to the vernal equinox, *Winter.*

It was believed for many centuries that these four intervals divided the entire length of the year into equal portions. But observation has caused differences to be discovered, which are themselves again found to be sensibly different at very distant epochs. This fact could be established by help of the gnomon, such as we have described it; and it was by analogous means that it was first detected. But the instruments now employed are incomparably more perfect, and these will enable us much better to appreciate the existence of the differences in question, as well as to estimate their precise value.

46. If through the point C (fig. 21) we draw in the meridian an indefinite line $P'CP$ perpendicular to the meridian, trace CQ of the plane of the equator, which we have just constructed, and if we fix a long hollow tube in this direction, the visual ray which coincides with the axis of the tube, towards the north, will mark in the heavens the apparent centre of the circle described by the stars which remain

constantly above our horizon. So that generalizing this result we may say that the diurnal revolution of the whole heavens appears to take place around the line $P'CP$, a circumstance which has given the name of *poles* to its extremities. It does not signify from what point on the earth's surface this line is drawn, the visual ray drawn from the observer's eye always appears to be the axis of the general rotation of the heavens. We shall see presently that the relative positions of the visual rays, drawn to a fixed star, during the various phases of its diurnal revolution, are rigourously conformable to this statement. For the moment let us confine ourselves to following out its consequences as regards the sun, neglecting the diurnal displacement which that body undergoes, in the direction of the meridian, in consequence of its proper motion while it is above the horizon. Now if, through the centre C of the gnomon, we conceive the straight line $P'CP$, figs. 23 and 24, to be drawn perpendicular to the equator CQ, the central solar ray σC, which passes through the aperture C, will describe each day round this line a right cone on a circular base, the semivertical angle σCP remaining constant in all the positions of the body. This angle is called the *North Polar Distance* of the sun's centre. The same ray σC produced towards the floor will describe the opposite sheet of the same cone. The section of this sheet by the horizontal plane, drawn through G, will be the curve which the luminous image will trace upon the floor during that day. This curve then will be a conic section, which in the particular arrangement represented in the figures will be a branch of an hyperbola, concave towards the point G when the sun is to the north of the equator, and convex when it is to the south; as may be seen at once from the figures, in which these two positions are represented separately and indicated by the shaded sheet of the cone. Between these two extremes the luminous image will describe a straight line when the sun is in the equator. All this is conformable to the general appearances which have been indicated in fig. 17. These results of course are not rigourously true, in consequence of the sun's proper motion; they may be considered as exact at the time of the solstices; the determination of the actual curve traced out by the luminous image at other seasons is a problem susceptible of mathematical solution, but upon which we shall not here enter.

47. Neglecting then the sun's proper motion which is insensible at the solstices, if we take upon the diurnal luminous curve, fig. 17, two points Σ Σ_2, at equal distances from G, we find that the angles ΣGN, $\Sigma_2 GN$ are equal. This affords an excellent means of tracing the meridian line; for we have only to describe round G as centre any number of circles of different radii, and to mark upon each of them the points in which it is cut in the same day by the centre of the solar image. Each pair of corresponding points thus obtained will give the direction of the meridian, by bisecting the chord which joins them; and if there be any difference between the lines so traced, the mean of them all will give the direction very accurately. Moreover, if we conceive the luminous rays which terminate in the points Σ Σ_2 to be produced to the south of the zenith, they will evidently have equal zenith distances, and will lie in planes equally inclined to the meridian, which shews that the diurnal arc described by the sun is divided into two symmetrical portions by the latter plane; saving the small dissimilarity produced by the proper motion towards the north or south in the course of the day.

48. After having operated thus at a certain point C, fig. 25, let us suppose that we trace on the horizontal plane the line EW perpendicular to the meridian at that point; and that in the vertical plane containing EW we take, towards the east for example, another point C' for which we perform the same process; then another C'' still further on, and so on proceeding constantly eastward. It is evident that the earth being spheroidal, this construction if continued will give us a re-entering curve $CC'C''C'''$...for the section of the earth's surface; and, according to the observation lately described, the axis of rotation of the heavens will always seem to pass through the point C, C', C'' where the observer is actually placed. Now this identity of appearance cannot be unless the diameter of the earth be as nothing compared with the distance of the observed heavenly bodies. And this result is necessary even if the diurnal revolution of the heavens be only an appearance produced by an actual rotation of the earth in the contrary direction, which we have seen to be very probably the case. This consequence will be more rigourously confirmed afterwards by observations sufficiently precise to indicate to us the distances of the heavenly

bodies which are nearest to us; and then we shall also discover certain minute differences in their appearances, depending upon the position of the places from which at the same moment we observe them. But if the dimensions of the terrestrial spheroid are not altogether inappreciable in comparison with the distance of these nearer ones, they are absolutely insensible relatively to all the others which constitute the great multitude of the heavenly bodies.

49. This explains very simply *why* the two solstitial rays $C\sigma$, $C\sigma'$, fig. 21, drawn in the plane QCP which contains the axis of diurnal rotation, make with each other an angle $\sigma C\sigma'$ which is found to be sensibly constant, from whatever point the observations are made. Let us represent by PP', fig. 26, the direction of this imaginary axis, which must always pass through the earth and cut it in two opposite points p p', whether it be the earth or the heavens which actually revolve. At any point C, whether at the earth's surface or in its interior, let us fancy an observer who at any instant measures or determines the north polar distance of the sun's centre σCP. If the same observation be made at the same instant from any other point C' also belonging to the earth, the smallness of the interval CC' as compared with the distance of the sun will render the rays $C\sigma$, $C'\sigma$ sensibly parallel; and the same cause will also render sensibly parallel the two lines CP, $C'P$ which are directed from the points C, C' to the apparent pole of the heavens P, round which the diurnal rotation appears to take place. Thus the angle σCP will be sensibly equal to $\sigma C'P$, provided that they be observed at the same instant. The same thing will hold of every other north polar distance, $\sigma' CP$, $\sigma' C'P$, under the same condition of simultaneity of observation. Now this condition is of necessity fulfilled when σCP, $\sigma C'P$ are the two solstitial distances, whether the greatest or least, provided that each observer has determined them exactly by their character of maximum or minimum, a thing so much the more easy to accomplish because, in these two positions of the sun, the polar distance σCP or $\sigma' CP$ does not perceptibly vary. Hence these two distances and their difference $\sigma C\sigma'$ or $\sigma C'\sigma'$ ought to have sensibly the same value; and the straight lines QC, QC', which bisect these angles, ought also to appear every-

where perpendicular to the apparent axis of rotation of the heavens, if they do so at any one place; for they are all parallel to each other. By a necessary consequence all the planes drawn through these straight lines CQ, $C'Q$ perpendicular to the axis of rotation are parallel; and on account of the small size of the earth, they will if produced meet the same stars, thus coinciding with one plane which we have already called the plane of the celestial equator.

This constant parallelism of the straight lines QC, QC' in space, gives them necessarily unequal inclinations QCZ, $QC'Z'$ to the verticals CZ, $C'Z'$ of the different places; and the magnitude of these inclinations determines the mean obliquity of the solar rays upon the horizon of each plane, which is the most important element of the annual temperature. This consideration, coupled with the facility of determining the angle QCZ by observations of the solstices, has caused it to be employed from the most ancient times as an element of relative position for different places, and it has been called the *geographical latitude.* From the very condition which determines it we see that *the geographical latitude of a place is the distance from the equator to its zenith, and that it is also equal to the angular altitude of the celestial pole, visible above its horizon,* taking the expression *altitude* in the astronomical sense which we have already attached to it, § 34. The points of the earth for which this angle is zero determine upon its surface a curve line which we call the *terrestrial equator;* and the two points p p' for which the angle is a right angle are called the *poles of the earth.* At the Observatory of Greenwich the apparent altitude of the north pole is about $51^0\, 29'$. These definitions are given here for the purpose of being able to enunciate exactly many remarkable phenomena of terrestrial physics which will soon present themselves to us; and the method of observation by means of which the quantities themselves can be obtained with the greatest precision must be matter for consideration hereafter.

50. For many centuries astronomers had no other instrument than the gnomon, and besides employed it with much less precision than belongs to the method which we have described. The Greek gnomons were only a thin cone of

metal CG, fig. 27, erected vertically, and throwing a shadow upon a horizontal plane HG. Now in constructing the profile of this shadow, in the vertical plane which contains the centre of the sun's disk, as represented in the figure, it is evident that its limits are very imperfectly defined. For let $\sigma'CR'$, $\sigma''CR''$ be the two extreme luminous rays, which coming respectively from the highest and lowest points of the disk, just graze the apex C of the cone CG, which in this application of it is called the *style*. The space $R'G$ will receive absolutely no light at all; for if from any point X in that space we draw a line to the summit of the style, it will meet the heavens in a point above the disk; and from the point R' such a line will just meet the disk in its highest point. The space $R'G$ then will be in *pure shadow*. But from points beyond R' the sun will begin to be visible; and a larger portion of the disk will be exposed to them as we go further from R', until we come to a point R'' from which the whole disk will be seen; after which it will be of course equally visible from all points beyond. We shall have then from R' to R'' a progressive diminution in intensity of the pure shadow, which is called in optics the *penumbra*, the pure shadow being called the *umbra;* and there will be *total illumination* everywhere beyond R''. Now it will be impossible, in these variations, to know the point Σ where the ray falls from the centre of the disk; it will even be very difficult to distinguish the point R' at which the umbra terminates. But suppose that we are obliged to fix it, as the ancient astronomers who used this apparatus seem to have done, the resulting zenith distance will be $\sigma'CZ$, that is to say, the zenith distance of the *upper limb* of the sun, instead of σCZ, which is that of the centre. So that in order to obtain this latter, which is the real index of the position of the body, it is necessary to add to the zenith distance, as determined by the limit of the umbra, the angle $\sigma'C\sigma$, that is to say, half of the angle subtended by the disk, an angle which is called the *sun's apparent diameter*, and which is about 32′. This reduction then of 16′ is necessary for all ancient observations made with a gnomon thus arranged; and yet Ptolemy himself has not noticed the error which arises from it. The first astronomer who avoided it was Cocheouking, who in 1271 employed the very simple modification of the *aperture gnomon*, which gives the true zenith distance of the sun's centre.

The most ancient observation with the gnomon which has come down to us from the Greek astronomers, is that which Pytheas made at Marseilles, about the year B.C. 350, to determine the distance of the sun from the zenith of that place, at the two solstices, in order to ascertain its geographical latitude. But we have analogous observations made in China, with the style-gnomon, by the prince astronomer, Thcheou-kong, which date much earlier, even to B.C. 1100*. We have also proof that the style-gnomon was known in Egypt, at a period still more remote; and that it was even employed under more complicated forms than those which have been here described. For, among the antiquities preserved in the Museum of Turin, the late M. Champollion discovered one, found at Thebes amongst tombs the date of which goes back as far as B.C. 2000, which offers all the characteristics of a style-gnomon, in which the style is oblique to the vertical plane, as in the actual sundials which are placed to mark the time of noon in public places. It is represented in fig. 28. It consists of a square pedestal of basalt, admirably polished, two of the lateral faces of which are elaborately sculptured. One of these has upon it two narrow parallel strips of stone, left in slight relief on the sides of the centre of a hole, in which probably the style was inserted; and between them, beginning at the centre of the hole, is traced a straight line of extreme fineness, to receive the shadow of the style. As a confirmation of this view of its destination, the contiguous face exhibits a representation of the sun-god Phre, turned towards the side from which the light comes. The base of the pedestal is prolonged parallel to the face exposed to the sun's rays, but this portion is now broken; and an hieroglyphic inscription sculptured upon the circumference of this base expresses that the instrument belonged to one of the priests, called *hierogrammats*, who were charged with the scientific studies relating to religion. Moreover, Clemens Alexandrinus bears testimony that this class of priests carried, in the public ceremonies, something similar to what has been above described, as the symbol of their functions†. The smallness of this specimen would seem to indicate that it was destined for such a use rather than for actual observations. But by giving to the apparatus larger dimensions, it would serve perfectly well for

* *Connaissance des Temps*, 1811, p. 429, &c.

† *Stromata*, Book VI. Chap. IV.

azimuthal observations, analogous to those of fig. 11, § 32; the projecting piece (now broken) indicating the direction perpendicular to that in which the sun rose when the shadow of the style fell on the vertical line; and nothing else would be required to trace horizontal meridian lines, which it appears that the ancient Egyptians very well knew how to do, since many of their buildings, and especially the pyramids, have their faces set very approximately, according to the direction so determined.

51. It is a very remarkable fact, that Ptolemy, who speaks frequently of instruments placed in the plane of the meridian, and who says that he had made observations in that manner himself, gives no indication whatever of the manner in which he determined the meridian line. Nor does Theon of Alexandria, who wrote a very extensive commentary upon the Almagest, give any explanation upon this point. The only mention, with which we are acquainted, of an operation so important for exact astronomy, is found in a work of Proclus Diadochus, entitled Ὑποτυπωσεῖς, the purpose of which is to explain the hypotheses necessary to represent the motions of the heavenly bodies, especially those of Ptolemy. In describing one of the principal instruments of the Almagest, which ought to be fixed in the plane of the meridian, the author teaches how to draw a meridian line. For that purpose, he directs us to trace a circle round the foot of a style-gnomon; then to mark the points in which the extremity of the shadow meets the circumference of this circle in the morning and in the evening, and to bisect the angle included between these two equal shadows. This process is evidently very imperfect. First, because the proper motion of the sun in the direction of the meridian makes the equal shadows to be unequally distant from the exact meridian, except at the epochs of the solstices; and, secondly, because the extremity of the shadow of a style-gnomon is always uncertain, on account of the penumbra, which destroys all possibility of limiting it with precision to equal lengths. One might hope to find more accuracy with the Arabians and Persians, who were acquainted with the Almagest before ourselves, and who have practised astronomy for a long time, according to its teaching; so that the processes of which they avail themselves give a very certain limit to the knowledge which was possessed

before them. Now up to the thirteenth century of our era, the most skilful and most practical of these astronomers, such as Ebn-Jounis and Aboul-Hassan, do not describe or allude to any other method of tracing a meridian line, than that by the equal shadows of a style-gnomon, as explained by Proclus. And we find, in fact, nothing more than this in the process described by them under the name of the *Indian Circle,* since it consists in erecting such a gnomon at the centre of a horizontal circle, and observing the points at which the extremity of the shadow enters the circle, and again quits it. All this tends to throw much doubt upon the spirit of precision which has sometimes been attributed to them, and also upon the exactness of the methods employed by their master Ptolemy; that is, if we are prepared to believe that Ptolemy had himself made observations, as he himself affirms, without however giving any details which tend to confirm his assertion. This entire absence of precise documents of observation, which we remark in the Almagest, might make us fairly suspect, as in fact Delambre believed, that Ptolemy was only a skilful mathematician, and an indefatigable calculator, who collected into one body the works of the previous astronomers, and specially those of Hipparchus; and whose original works, which would now have been so valuable to us, the Almagest, which became popular, unfortunately caused to disappear for ever.

52. The other instruments, not many in number, which Ptolemy speaks of as having been used by himself or by Hipparchus, have been reproduced by the Persians and Arabians, with additions in their details, in which the desire of the greatest precision in astronomical observations begins to shew itself in important improvements.

53. The successive returns of the sun to the plane of the meridian, or more generally to the same phase of its diurnal revolution, measuring out for mankind the alternate intervals of work and repose, have been universally employed, under the name of *solar days,* to determine the civil unit of time. But the subdivisions of this common unit have been very different with different people, according to the greater or smaller degree of precision which their social condition rendered it necessary to adopt.

We moderns who possess the mechanical instruments called *clocks*, which divide time into portions exactly equal, independently of the presence or absence of the sun, and which make these divisions known to the public, subject the whole diurnal revolution of the sun to one common and uniform system of subdivision, without distinction of the phases of light and darkness. For that purpose we conceive twelve indefinite planes, called *horary planes*, to be drawn through the axis of diurnal revolution, which make with each other angles of 15^0; so that all the celestial space which surrounds this axis is thus divided into twenty-four equal segments, each comprising one of these angles. Then we denote by the name *solar hour* the time, whether constant or variable, which the sun employs each day in passing from one of these planes to the next. One of them is always the meridian; and the succession of hours is reckoned from the instant of the sun's centre passing it in the part below the horizon, which constitutes what we call *midnight*, whilst we denote by *midday* or *noon* the instant of the centre crossing the part above the horizon. In astronomy the twenty-four hours which compose each solar day are reckoned continuously from one to twenty-four, from one midnight to the following midnight. But in civil usage, the custom prevails of breaking off the reckoning after twelve hours, that is to say, at noon, and of recommencing; distinguishing the hours which precede noon by the term *morning* or *forenoon*, those which follow by *evening* or *afternoon*. Our clocks, public as well as private, usually mark the hours according to this system of reckoning.

The Chinese, who, more than twelve centuries before the Christian era, employed a mechanical method for the subdivision of time analogous to our own, but not so perfect, applied it to the whole diurnal revolution of the sun without discontinuity. But they divided it into a hundred equal portions, which were successively made known to the public. They were reckoned like our own from midnight. They had also another mode of subdivision into twelve parts called hours, which they employ still, and which is in fact the only one now in use. But there is no evidence that the employment of this method dates back to such antiquity.

The Greeks, and even the Arabians, up to the epoch when they become acquainted with mechanical methods of measuring time, divided the solar day differently. They divided the duration of the visible day into twelve equal parts, called *day-hours,* and that of the night into twelve others, called *night-hours.* These two kinds of hours therefore varied in length for the same place, at different seasons; and they were also different at different places at the same epoch of the solar year, according to the manner in which the entire revolution of the sun was divided by the two phases of night and day. It was only at the equinoxes that they became equal for the night and the day, and at the same time common to all countries. On this account such hours have been called *temporary hours;* while we denote by the name of *equinoctial hours* the equal hours which are exclusively used in these days.

54. In the absence of clocks to measure these variable hours, the solar shadows thrown by the summit of the styles of gnomons were used, the point corresponding to each hour being marked upon the diurnal curves traced out by these shadows.

55. These instruments depending upon solar shadows, imperfect as they were, have still served to establish, that the proper annual motion of the sun, from whatever place it is observed, appears always to take place in a plane oblique to the celestial equator, and drawn through the eye of the observer; this last circumstance resulting from the fact that the earth is merely like a point, as compared with the distance of the sun.

In order to draw this deduction, it is necessary to remember that the continual displacement of the sun amongst the fixed stars exhibits it to us as proceeding always from west to east, by a proper motion directed obliquely to the equator and to the meridian, until it returns at length to the same position in the heavens, after a number of solar days, which we have ascertained to be approximately equal to $365^{d}.25$. This annual motion and return can therefore be represented geometrically, by attributing to the sun two simultaneous and distinct motions, one of which parallel to the equator carries it always eastward, and the other in the direction of the horary planes is measured by the diurnal variations of meridian zenith distance, which can be concluded from the gnomon. Further, the motion parallel to the equator may be represented approximately, if we conceive a system of horary planes to be drawn through the axis of diurnal rotation making with each other angles each equal to $\frac{360^0}{365.25}$, or $1^0 - \frac{7}{487}1^0$, and consider the sun to be transported from one plane to the next in the interval of two consecutive noons. This being understood, take arbitrarily a point C, fig. 29, to represent the place on the earth's surface from which the observations are made. Through this point draw the axis PCP', round which the diurnal revolution of the heavens takes place, also the plane $EQ\,E'Q'$, perpendicular to this axis, to represent the equator. If with C as centre we describe a sphere of any radius, which cuts the polar axis in the points $P\ P'$, and the equatorial plane in the circle $EQE'Q'$, the visual rays drawn from C to the sun, as well as every other heavenly body, will be cut by this sphere; and the angles included between the rays will be measured by the intercepted arcs of great circles on its surface. Now it is very easy to construct graphically upon this surface the successive points on which the sun is projected at noon. In fact, having constructed a material sphere, mark upon it arbitrarily two points $P\,P'$ at the extremities of a diameter, to represent those in which it is intersected by the axis of diurnal revolution; and trace a great circle perpendicular to this diameter, to represent the intersection of the sphere with the plane of the equator. Then let us agree to mark upon the surface the *apparent place of the sun* at noon, beginning with a certain day for which we have measured the meridian zenith distance σCZ fig. 21, by observation with the gnomon. If we have also determined, as we are able to do, the constant distance ZCP of our zenith from the visible pole, which is the complement of the apparent and observable altitude of that pole, we shall at once conclude what was the north polar distance of the sun σCP at the moment of observation. To construct it we shall describe upon the sphere, fig. 29, a great circle Pq' passing through the pole, which will represent the trace of the horary circle, in which the sun was at noon on the day in question. Then taking upon this circle an arc $P\sigma'$ equal to the observed polar distance, the extremity of the arc σ' will represent the apparent place of the sun on the sphere at that moment. The next day the operation must be repeated; only it will be necessary to construct the new polar distance upon another horary circle Pq'', the plane of which makes with that of Pq' an angle equal to $1^0 - \frac{7}{487}1^0$, *towards the east.* Nothing can be more easy than this, since the angle will be measured and expressed by the corresponding arc of the equatorial circle. We shall then have therefore, upon the sphere, the place of the sun for the second day at the instant of noon. We can obtain it in like manner day after day, as often as we can make observations. Then drawing a continuous curve through all these isolated points, we shall see that it is sensibly a great circle of the sphere. So that, to judge from appearances, the annual course of the sun takes place in the plane of this circle, which passes through the centre C, that is through the eye of the observer.

56. This construction by points of the polar distances of the sun upon a divided globe, to represent the annual motion of that body, appears to have been adopted by the Chinese at epochs at least as ancient as the gnomon observations of Tcheoukong. And indeed the division of the circumference, used by them from time immemorial, seems connected with it. For instead of dividing it into an integral number of degrees, as other nations do, they made it, and do make it still, to consist of 365 parts and $\frac{1}{4}$, conformably to the number of days and fractions of a day which they found, or supposed, to be contained in the solar year. So that a Chinese degree is equivalent to $\frac{360^0}{365.25}$, or to $1^0 - \frac{7}{487}1^0$, or $1^0 - 0^0.01437371663$. And they supposed the sun to describe, between each noon and the succeeding one, parallel to the equator, one of these degrees exactly, precisely as we have supposed in our construction.

57*. [Independently of the practical imperfections which always attach to a graphical construction, this method of representation involves an element essentially doubtful and hypothetical, which is the constancy of the motion attributed to the sun parallel to the equator, in the interval of two noons, throughout the year. But here, as in an infinite num-

[* The Student who is not acquainted with Spherical Trigonometry may omit this Section.—Ed.]

ber of scientific investigations, the simplicity of the physical law which seems to manifest itself is an indication that we must examine and see whether, on the supposition of its truth, it would not offer some means of removing this uncertainty by its observable consequences. Suppose, then, that the annual path of the sun in the heavens lies wholly in one plane, drawn through the centre of an imaginary sphere, and which consequently cuts it in a great circle $ESE'S'$. This plane will cut the plane of the equator in the line ECE', which will also pass through the centre C; so that CE', and CE will be the directions of the visual rays drawn to the sun in its two equinoctial positions in the same year. These rays then, being already constructed upon the equatorial circle, ought to be found to be in the same straight line. Now, through C let us draw a third plane PSQ, perpendicular to the intersection ECE' of the two others, and let SCS', QCQ', be the lines in which it intersects those two planes. The angle SCQ, or $S'CQ'$, included between these straight lines will measure the inclination of the two planes. Thus, CS, CS' will be the visual rays directed towards the sun at the periods of its greatest distance from the equator; and the equality of the opposite angles SCQ, $S'CQ'$, shews that these distances are equal. These, then, will be the true places of the solstices. We should have, therefore, the exact values of these angles, by taking the complement of the least or of the greatest polar distance observed throughout the year, provided that the sun crossed the meridian of the place of observation at the moment of arriving at the solstice. Evidently this coincidence cannot be realised accurately at every solstice except for certain places on the earth's surface. Nevertheless, we can obtain the angles SCQ, $S'CQ'$, almost without any error, by deducing them from the two extreme polar distances, observed at any place, because the solstitial polar distances PS, PS', being perpendicular to the plane $SE\ S'E'$, differ in only an indefinitely small degree from the neighbouring values. Now having thus the inclination (ω) of the two planes, we can, by the help of spherical trigonometry, place upon the sphere the arcs $q'\sigma'$, $q''\sigma''$,... QS, which shall be the complement of the polar distance for each day, without making any supposition concerning the arcs $q'q''$, $q''Q$, of the equatorial circle, which separate them*. In fact, E'

* There is a passage in the commentary of Hipparchus on Aratus, in

representing one of the points of intersection of the two circles, which we will suppose to be the vernal equinox, let us conceive that we have determined a certain meridian distance of the sun from the equator $q'\sigma'$, which has been observed between that equinox and the summer solstice; then let it be required to determine the distance from the intersection E' which the inclination of the planes assigns to it. The arcs of great circles $q'E'$, $\sigma'E'$, $q'\sigma'$, will form a right-angled spherical triangle, having the right angle at q', in which we know the angle at E' which is the inclination of the two planes, or ω, and also the distance $q'\sigma'$ which is given by observation, and which we will call d. Therefore all the other parts of the triangle may be determined. If we require, for instance, the arc $q'E'$, which we will call a, and the arc $\sigma'E'$, which we will call l, they will be found from the formulæ,

$$\sin a = \frac{\tan d}{\tan \omega}, \quad \sin l = \frac{\sin d}{\sin \omega}.$$

Each of the arcs a being thus concluded from the distance d which corresponds to it, it can be constructed upon the sphere, measuring from the point which has been chosen to represent the intersection E', without making the supposition that the difference of the arcs a between two successive noons is constant throughout the year, as we supposed it to be before. Hence, if this difference varies at different epochs we shall obtain by this means the true law of its variation. It will remain then only to devise some method of observation, which shall give us the measure of these successive

which this great astronomer asserts distinctly that in a treatise of his, now lost, he has *demonstrated by figures*, relations of the arcs analogous to those treated of in the text; and, in fact, this very commentary contains an example much more difficult which is strictly calculated. As we find no trace of spherical trigonometry in the earlier Greek works which have come down to us, nor in the results which they have obtained, I conclude with Delambre that we must attribute to Hipparchus the invention of this branch of mathematics, which is so valuable for the study of Astronomy. The Chinese, who practised Astronomy many centuries before the Greeks, became acquainted with spherical trigonometry only at a very late period. Cocheouking himself received it in the thirteenth century of our era from the Persian astronomers; and by an inaptitude for exact notions which we find so frequently in this singular people, he made errors in many very simple applications of the science, although he shewed himself so careful and skilful in the practical operations of observing.

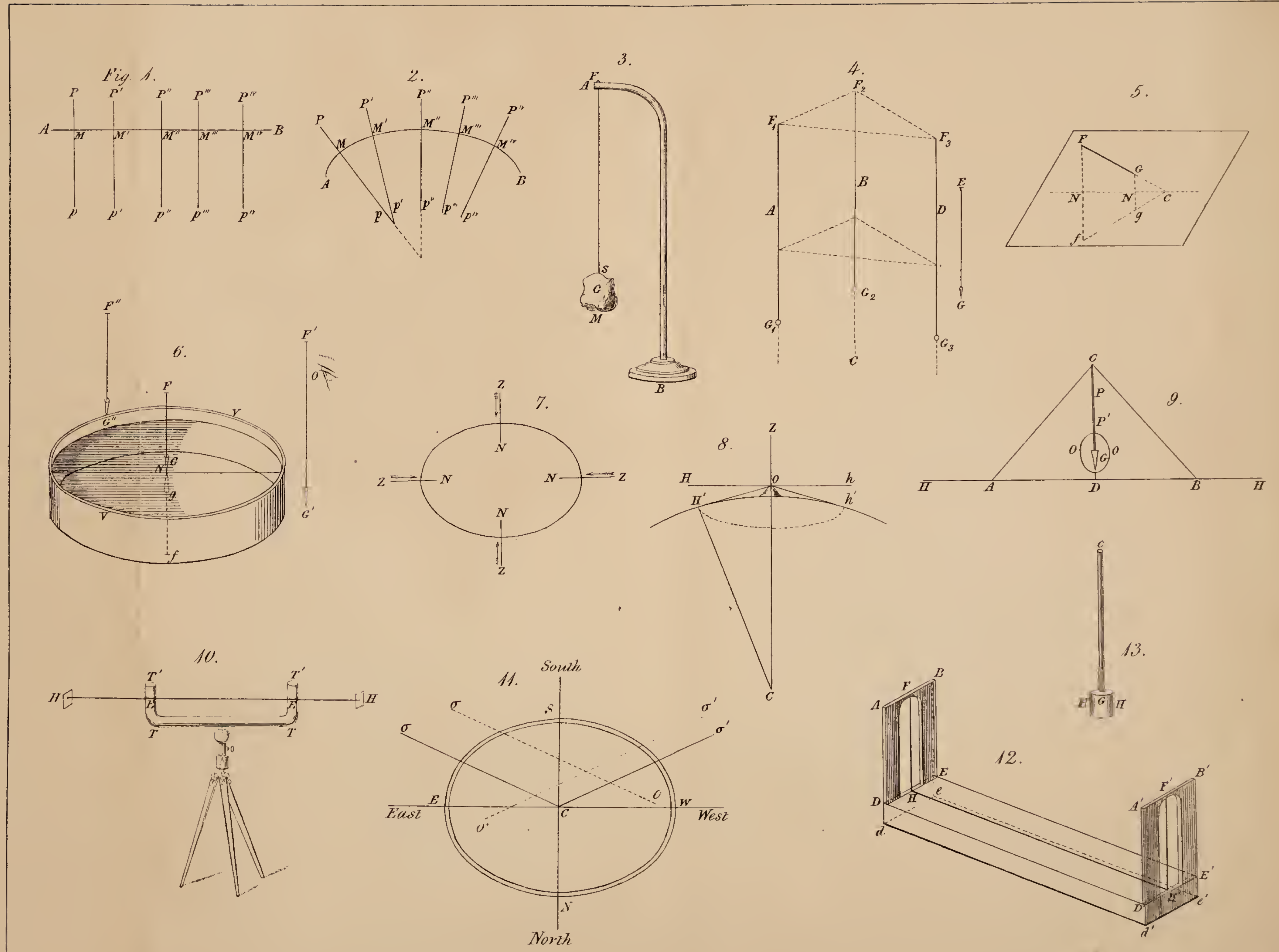
differences. For if we find them always agreeing with the results of calculations, throughout the whole of the equatorial circle, the condition of the sun's motion in one plane will thus be confirmed by each observation, and the combined force of all these verifications will put it beyond a doubt.]

58. The arcs $q'E'$, $q'\sigma'$, which we have been just now considering, are in reality the quantities which determine at each instant the apparent position of the sun relatively to the equator, and to a fixed horary plane supposed to be drawn through the vernal equinox. The first, $q'E'$, or a, is called the *right ascension;* and it is reckoned from the vernal equinox, through the summer solstice, from 0^0 to 360^0. This mode of reckoning makes it follow the motion of the horary plane which contains the sun; and it increases thus from west to east, in proportion as this plane advances in that direction. The arc $q'\sigma'$, or d, which is the complement of the polar distance, is called the sun's *declination.* It is the measure of the angle which the visual ray drawn to that body makes at each moment with the plane of the equator. As the direction in which it is measured changes in the course of the year, according as the sun is on the north side or on the south side of the equator, we affect the expression d with opposite signs to indicate this opposition; and we usually characterize northern declinations by the sign +, and southern by −. If we make use of the north polar distance instead of the declination, we have no convention of this kind to attend to. The arc $E'\sigma'$, or l, which completes the spherical triangle, and forms its hypothenuse, is called the sun's *longitude.* It is reckoned, in the same manner as the right ascension, from the vernal equinox, and in the same direction, from 0^0 to 360^0.

59. The plane which contains the sun's annual path is called the *ecliptic,* because it is in some sense the place of eclipses; for the moon cannot hide the sun from us, or penetrate the earth's shadow, except when it is very nearly in this plane, or actually in it, and upon the indefinite straight line drawn through the sun and the earth; whether it be in *conjunction,* that is to say, on the same side as the sun, or in *opposition,* that is to say, on the opposite side.

60. To complete these definitions, it may be added that in astronomical calculations, the sun is usually denoted by the symbol $\odot$, which appears to be of very high antiquity; for we find it employed for this purpose upon the most ancient monuments of Egypt, and among the most ancient Chinese characters.

61. We see from the preceding discussion that the fact of the annual motion of the sun in one plane passing through the eye of the spectator, cannot be completely demonstrated by observations with the gnomon alone; since these give only one of the quantities necessary to determine from day to day the position of that body, namely, the declination or north polar distances, and that the other quantity, namely, the right ascension, has been only deduced from the hypothesis of a uniform motion parallel to the equator, or from the supposition itself of the motion taking place in one plane. We require then some means for determining day by day this second quantity, the right ascension; and we can arrive at it only by a comparative measure of the intervals of time which pass between consecutive noons. We have in these days, in our *clocks*, very exact means of determining these intervals, means which were unknown to the ancients, or the place of which they were able to supply only by very imperfect processes. We must then devise instruments of such delicacy as to enable us to get rid of all such imperfect methods. We must also give a corresponding degree of accuracy to our measures of zenith distance, an accuracy which cannot be attained with the gnomon, which moreover is only applicable to observations of the sun. This brings us to the processes of modern astronomy, which is entirely distinguished from the ancient by the spirit of precision which characterizes its instruments of observation, its methods of calculation, and its mechanical theories.



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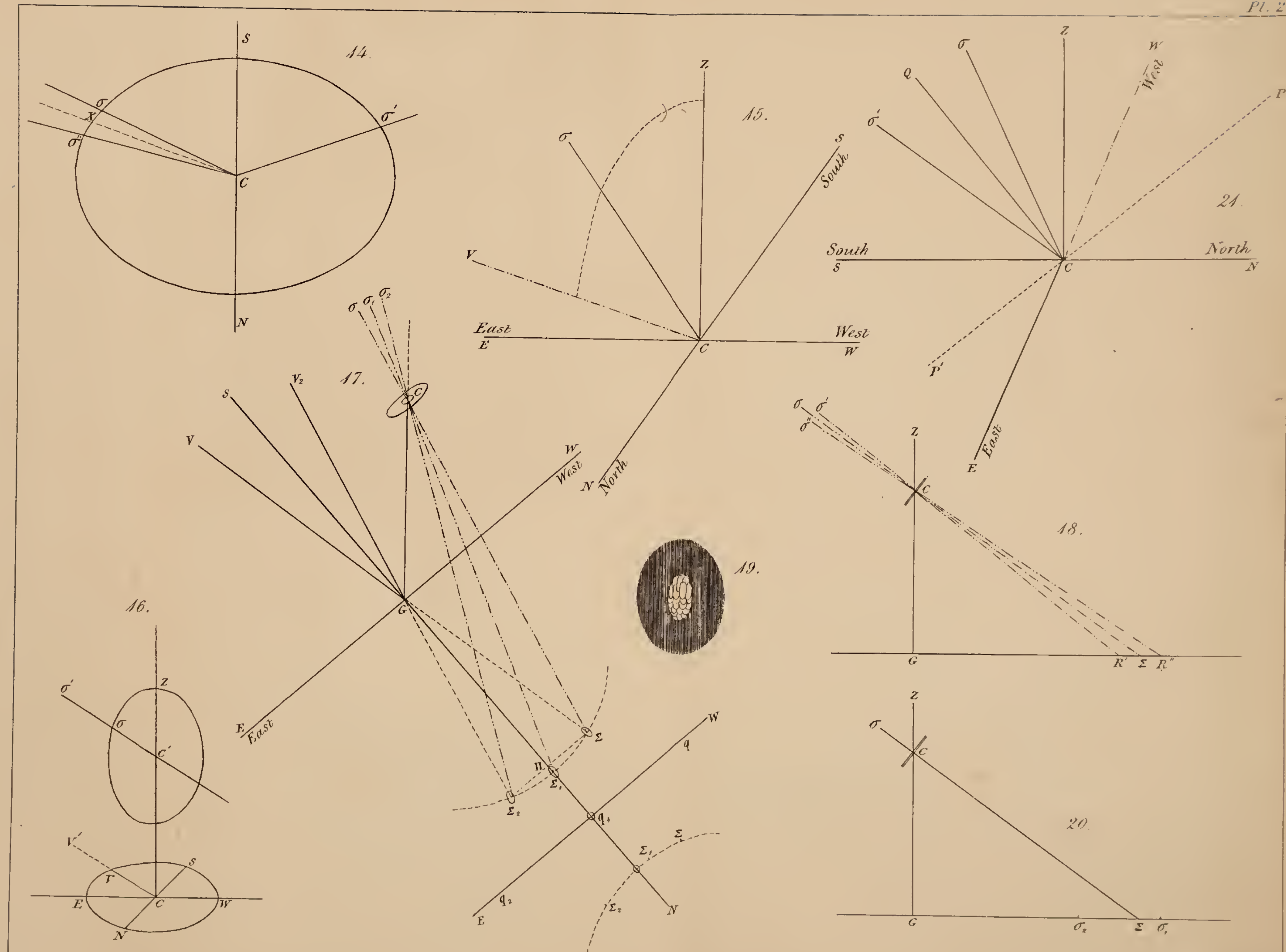

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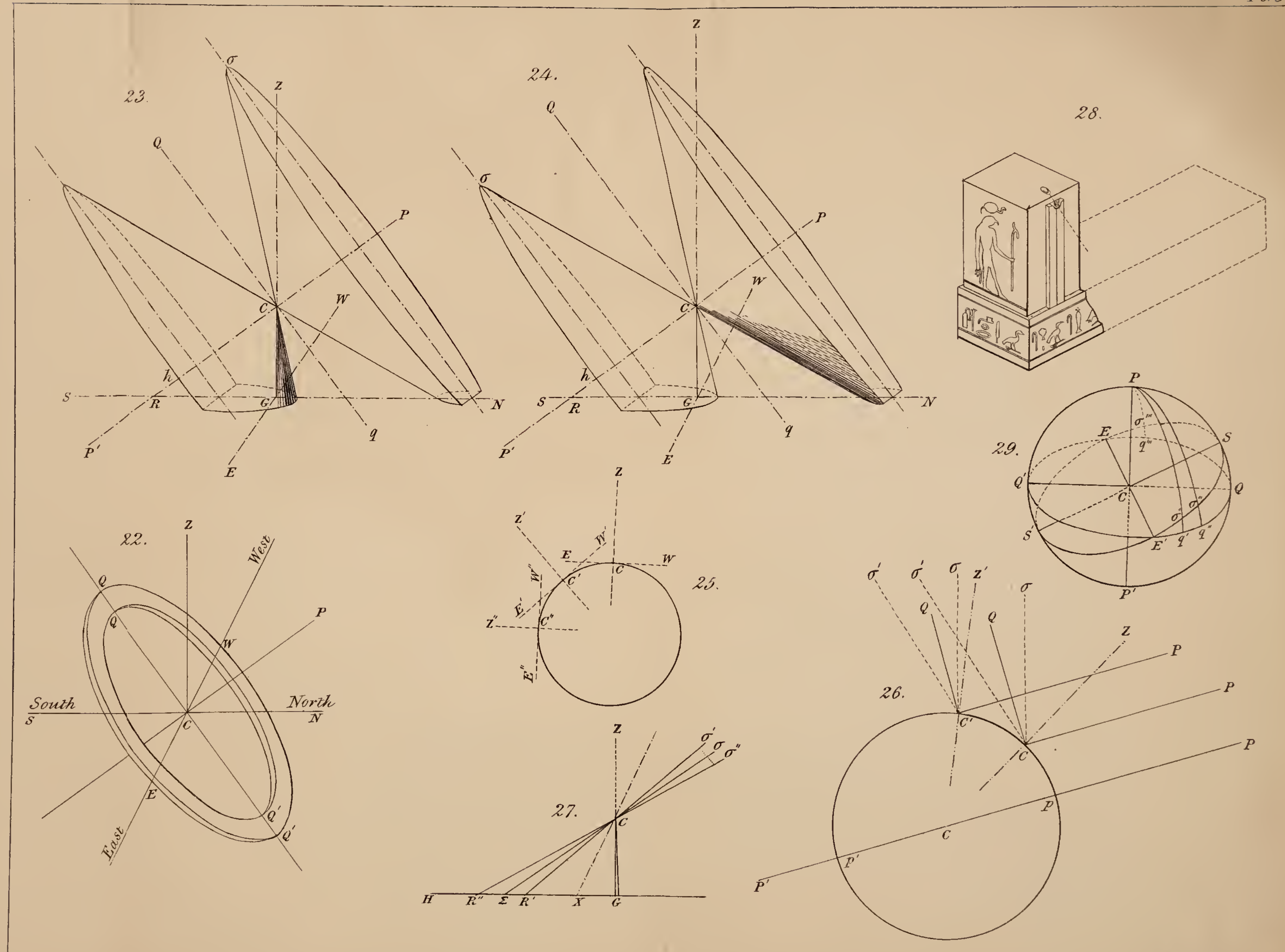

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